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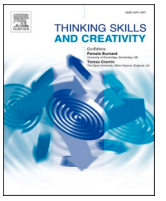

Disjunction and access to knowledge: Educational implications

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ABSTRACT

In classical logic, it is possible to derive 'either p or q' from 'p' (where p and q are sentences with any content). This is a cognitive problem, since people often tend not to make inferences of that kind. This paper analyzes the solution the theory of mental models gives for this problem. Based on that solution, the paper proposes to use tasks with inferences such as that mentioned in the educational context. The idea is that those tasks can allow assessing certain learnings. In particular, they can reveal in some cases whether students understand the relations between the semantic contents assigned to p and q. This is because, following the theory of mental models, whenever q implies p, the inference must be accepted.

1. Introduction

The development of people and society is important. That is the reason why societies made the decision to formalize the place where we should form ourselves: school. From scientific progress in different fields, there are several teaching theories and methods. Those theories and methods are based on strong evidence, and they try that students learn in the best way. In the complex teaching-learning process, there is a relevant component that is not always acknowledged and valued: learning assessment. Nowadays, there is a consensus about the fact that assessment can condition learning. It is important what is assessed and how the assessment is made. This applies to both summative and formative assessment. The latter is focused on the support during the formative process, in which the key is the particular feedback. The former addresses results of learning (see, e.g., Förster, 2023).

The aim of the present paper is to analyze, based on the theory of mental models, whether the disjunctive inferences can be suitable to assess particular learnings. The paper will have four sections. The first one will account for logical connectives and the case of disjunction (1.1.). The second section will explain why, from the logical point of view, it is correct to infer a sentence such as (2) below from a sentence such as (1) below (1.2.). The third section will describe how the theory of mental models understands disjunction and why its account can solve this disjunction problem (1.3.). Finally, the fourth section will propose the idea of using disjunctive inferences of the kind considered as assessment instruments (1.4.).

1.1. Reasoning and logical connectives: the case of disjunction

Reasoning implies to make inferences from premises. It is usual to differentiate two kinds of reasoning: inductive and deductive. Inductive inferences are probabilistic, while deductive inferences are necessary true conditions. This paper will deal with the latter type. The basis to know whether a deductive argument is valid has been classical logic in several reasoning theories. Some procedures

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to decide whether an argument is correct have resorted to propositional logic. In this logic, the formal structures of the relations between propositions (which can be true or false) are crucial. Propositions are linked by means of the logical connectives: 'and', 'or', 'if…then', and 'if and only if' (see, e.g., Espino, 2004).

Connective 'and' is called 'conjunction'. It is false if one of the conjuncts is not true, or both conjuncts are not true. Connective 'if… then' is named 'conditional'. It is only false when the first clause, that is, the antecedent, is true, and at the same time, the second clause, that is, the consequent, is false. 'If and only if' is the biconditional, which is false when one of the clauses, not both of them, is false. Lastly, 'or', the connective that will be dealt with here, is known as disjunction. 'Either…or…, or both of them' is an inclusive disjunction. It is false only when its two disjuncts are false. On the other hand, 'Either…or…, but not both of them' is an exclusive disjunction. This change leads the disjunction to be also false when both disjuncts are the case (see also, e.g., Restall, 2006).

Disjunction is crucial for recognizing different options and, simultaneously, ruling them out. Despite the profound lack of knowledge regarding the possible preverbal logical abilities of babies, recent studies have begun to focus on disjunctive reasoning (Cesana-Arlotti & Kovács, 2020). While there may not be complete certainty regarding the understanding of disjunction, especially during childhood, it can be said that it begins to manifest at the age of 3 through speech, although exclusive comprehension of disjunction fails by the age of 5 (Grigoroglou & Ganea, 2022). However, it is also suggested that disjunctive reasoning does not depend solely on language experience, as it also arises in non-linguistic spatial tasks (Bohus, Cesana-Arlotti, Martín-Salguero & Bonatti, 2023).

But from the cognitive point of view, disjunction presents difficulties with older people. Logic allows building a disjunction from a fact taking that fact as one of its disjuncts (see, e.g., Deaño, 1999; Restall, 2006). For example, given (1),

(1) My pet is a dog

Classical logic enables to infer (2).

(2) Either my pet is a dog or my pet is a cat

This is a cognitive problem because people do not always make this type of inferences (e.g., Orenes & Johnson-Laird, 2012). The solution seems to be obvious: people do not derive conclusions following logic, or, at least, following classical logic. This last option was that of the theories such as the mental logic theory (e.g., Braine & O'Brien, 1998). Its proponents claimed that there was a logic in the human mind. However, that logic was not classical logic. The real mental logic included many elements from classical logic, but those logics were not identical. In fact, the mental logic did not allow the deduction of a sentence such as (2) from a sentence such as (1).

Nevertheless, this might not solve the problem. That solution would be suitable if people never inferred sentences such as (2) from sentences such as (1). Nonetheless, individuals sometimes make that kind of inferences (e.g., Orenes & Johnson-Laird, 2012).

The theory of mental models (e.g., Johnson-Laird, 2023, 2023; Khemlani, Byrne & Johnson-Laird, 2018) appears to be able to explain this phenomenon. It can describe the two situations: when people make the mentioned inferences and when they do not. However, the point of this paper is that the account of the theory of mental models is based on semantic criteria. According to the theory, the reasons why individuals make or do not make the indicated inferences mainly depend on their knowledge of the meanings of the disjuncts. So, this type of inferences can be an assessment instrument, since it can show to what extent people know definitions of concepts.

1.2. The disjunction problem and cognitive science

Classical logic allows deducing (2) from (1). This is because it allows deducing (4) from (3).

(3) p
(4) Either p or q

Where p and q are propositions with any thematic content.

The reason for this is not hard to understand. To be true, (4) only needs one of its disjuncts to be true. If p, which is one of the disjuncts, is taken as a premise, that means that p is assumed to be true. Thus, if p is true, one of the disjuncts in (4) is true. Therefore, (4) is also true.

As said, disjunctions can be either inclusive or exclusive (see, e.g., Espino, 2004; Khemlani, Orenes & Johnson-Laird, 2014). In the first case, p and q in (4) can be true at once. If the disjunction is exclusive, this is not possible. An exclusive disjunction is, for instance, (5).

(5) Either I am in Paris or I am in Brussels

Paris and Brussels are two cities. Hence, I cannot be in them at the same time. However, this has no influence on the problem this paper is addressing. The fact that I am in Paris is enough for (5) to be true.

If the disjunct that is true is the second one, that is, in (5), that I am in Brussels, this also suffices for the truth of the entire disjunction. No matter which the true disjunct is. Disjunctions only need one of them to be true. Thereby, if the premise is not (3) (i.e., p), but (6),

(6) q

It is possible to derive (4) (i.e., either p or q) from it too.
Something similar can be said regarding negations. If, for example, the premise is (7),

(7) Not-p

A conclusion such as (8) can be deduced.

(8) Either not-p or q

Likewise, for instance, (10) could be inferred from (9).

(9) Not-q
(10) Either p or not-q

The problem is that people do not usually make inferences such as those indicated above. One might think that this phenomenon is a piece of evidence that the human mind has no direct link to classical logic. That was the assumption of the mental logic theory (e.g., Braine & O'Brien, 1998). As mentioned, this theory did not reject the idea that there is a logic in the human mind. Its proposal was that human beings, when reasoning, follow many schemata from classical logic, but not all of them. One of the schemata that people do not consider is the one allowing making derivations such as those in this section.

Nevertheless, this does not solve the disjunction problem. It is not the case that individuals never make this kind of inferences. The case is that individuals make them few times. In some occasions, people make those inferences. Accordingly, a theory trying to explain the phenomenon should describe when inferences such as those mentioned above are made and when those very inferences are not made. The theory of mental models offers an account (e.g., Orenes & Johnson-Laird, 2012).

1.3. The disjunction problem and the theory of mental models

The theory of mental models understands connectives as conjunctions of possibilities (see also, e.g., Johnson-Laird, Quelhas & Rasga, 2021; Johnson-Laird & Ragni, 2024; Khemlani, Hinterecker & Johnson-Laird, 2017; López-Astorga, Ragni & Johnson-Laird, 2022; Rasga, Quelhas & Johnson-Laird, 2022). In the case of disjunction, this can seem counterintuitive. This is because disjunction and conjunction appear to be contrary. However, the theses of the theory of mental models resolve this apparent contradiction: if an inclusive disjunction can be true in three cases (when the two disjuncts are the case, when the first disjunct is the case, and when the second disjunct is the case), there are three possible situations in which, if inclusive, a disjunction such as (4) can be true. (11) presents those situations (see also, e.g., Johnson-Laird & Ragni, 2019).

(11) Possible (p & q) & Possible (p & not-q) & Possible (not-p & q)

The three possibilities in (11) represent three mental models (see also, e.g., Byrne & Johnson-Laird, 2020). They are the three mental models that individuals tend to think about when they find an inclusive disjunction. If the disjunction is exclusive, the possibilities or models are only two, since the first one in (11) disappears (see also, e.g., Quelhas, Rasga, & Johnson-Laird, 2019). In this way, the models of (2) (i.e., either my pet is a dog or my pet is a cat) are those in (12).

(12) Possible (my pet is a dog & my pet is not a cat) & Possible (my pet is not a dog & my pet is a cat)

The interpretation that (12) stands for is that in which (2) is exclusive, that is, in which I only have one pet. If (2) were an inclusive disjunction and I could have more than one pet, the models would be the following:

(13) Possible (my pet is a dog & my pet is a cat) & Possible (my pet is a dog & my pet is not a cat) & Possible (my pet is not a dog & my pet is a cat)

A sentence such as (5) (i.e., either I am in Paris or I am in Brussels) only allows an exclusive interpretation. Its possibilities are as follows:

(14) Possible (I am in Paris & I am not in Brussels) & Possible (I am not in Paris & I am in Brussels)

Thus, the models of (8) (i.e., either not-p or q) and (10) (i.e., either p or not-q) can be easily derived. If it is assumed that they are inclusive disjunctions, those of (8) are those in (15).

(15) Possible (not-p & q) & Possible (not-p & not-q) & Possible (p & q)

Those of (10) are indicated in (16).

(16) Possible (p & not-q) & Possible (p & q) & Possible (not-p & not-q)

There are more important points in the theory of mental models. For example, it is a dual-process theory such as those Evans (2008) describes. This means that individuals only manage to identify all the possibilities that can correspond to a sentence if they endeavor and think carefully (see also, e.g., Johnson-Laird, Khemlani & Goodwin, 2015). Nevertheless, the thesis of the theory that is more relevant for the goals of the present paper is that of modulation (e.g., Quelhas & Johnson-Laird, 2017). According to this thesis, semantics and pragmatics can have an influence on models (see also, e.g., Quelhas, Johnson-Laird & Juhos, 2010). For instance, unlike the case in which (2) (i.e., either my pet is a dog or my pet is a cat) is inclusive, (17) only has two possibilities.

(17) Either this animal is a cat or this animal is a feline

This is an inclusive sentence, since the animal can be a cat and a feline at once. Nonetheless, it does not have three models, but only those that (18) points out.

(18) Possible (this animal is a cat & this animal is a feline) & this animal is not a cat & this animal is a feline)

The missing model refers to an impossible situation: that in which this animal is a cat and not a feline. So, that possibility cannot be taken into account (for more examples akin to this one, see, e.g., Orenes & Johnson-Laird, 2012).

Following the theory of mental models, modulation is the phenomenon that explains the disjunction problem: modulation allows accounting for under what circumstances inferences such as that of (4) (i.e., either p or q) from (3) (i.e., p) are made and under what circumstances those very inferences are not made. If the premise is (1) (i.e., my pet is a dog) and the conclusion is (2) (i.e., either my pet is a dog or my pet is a cat), people will tend not to admit the inference. This is because, among the models of the conclusion, there is one incompatible with the premise. Both if (2) is exclusive and if it is inclusive, it refers to a possibility in which my pet is not a dog. That is what happens in both the second possibility in (12) and the third possibility in (13). In those cases, the scenario describes a situation in which my pet is not a dog, but a cat. That is inadmissible if the premise (1) is true. So, individuals will tend to deem inferences with these characteristics as incorrect.

But this cannot occur if the premise is (19)

(19) This animal is a feline

And the conclusion is (17). As (18) shows, (17) does not have any models inconsistent with (19). The animal is not a feline in no model in (18). Therefore, people should accept the inference in this case.

The predictions of the theory of mental models for the disjunction problem are clear (see for this point especially Orenes & Johnson-Laird, 2012) in several inferential schemata (see also, e.g., Johnson-Laird et al., 2021; Rasga et al., 2022). That has allowed checking those predictions by means of empirical research. The experimental results obtained are also clear. Orenes and Johnson-Laird's (2012) study showed the results relevant for the present paper. In their study, Orenes and Johnson-Laird carried out different experiments (with college students and random presentation) related to the action of modulation. Some of them addressed problems akin to those described above in conditional inferences. But the study interesting here is the second one.

The second study by Orenes and Johnson-Laird (2012) included disjunctive inferences in which the content did not eliminate the crucial possibility, and disjunctive inferences in which modulation removed that very possibility. The participants accepted more inferences in which modulation prevented the possibility contradictory with the premise (in the latter case, the percentage of acceptance was 79 %; when modulation had no action, the percentage was 29 %). This was one of the tasks of the study in which modulation blocked the problematic possibility:

> "Ana read a novel. Does it follow that Ana read Don Quixote or she read a novel?" (Orenes & Johnson-Laird, 2012, p. 375).

A task in which modulation did not have effect was:

> "Eva read a newspaper. Does it follow that Eva read Don Quixote or she read a newspaper?" (Orenes & Johnson-Laird, 2012, p. 375).

In summary, people infer a conclusion such as (4) from a premise such as (3) only in the cases indicated by the theory of mental models (see, e.g., Orenes & Johnson-Laird, 2012. The study and the experimental data described in the latter paper are so clear and relevant that the proponents of the theory refer to them from their publication on; for example, the paper is cited in recent works such as Johnson-Laird, Byrne & Khemlani, 2024). This can lead to propose an assessment instrument for concepts and definitions based on the framework the theory of mental models gives for disjunction. The next section explores this idea.

1.4. Educational assessment and the disjunction problem

To talk about educational assessment is generally to refer to the monitoring of learning processes. That requires significant time and resources, and it is a difficult challenge for teachers (Swiecki et al., 2022). For assessment to be effective, previous design of activities is

essential. It is necessary to take care of the coherence to what will be assessed and the way to assess (Ibarra-Sáiz, Rodríguez-Gómez & Boud, 2020).

In addition, at least two more contextual elements need to be taken into account when speaking about educational assessment. First, it is known that the new educational standards, the recommendation "No Child Left Behind", and the changing context that technology has created by means of the massive internet access and artificial intelligence pressure the educational system in different levels. They put, inter alia, active learning and assessment focused on the student at risk (Gredler, 1999; Jackson & Davis, 2000). Second, there are several diagnoses showing that teachers have a low development to assess in the classroom. Teachers do not have suitable training to assess learning. Their assessment is not generally valid, as it does not correspond to the learning (see, e.g., DeLuca & Bellara, 2013; De Luca, Willis, Cowie, Harrison & Coombs, 2023; Förster, 2023). This low level of knowledge about assessment not only is revealed in their command (technical and practical) of assessment. Many times, they do not know how to address relevant dimensions such as the socio-emotional one, which includes elements such as the anxiety/evaluation apprehension (Pastore & Andrade, 2019).

Currently, education has new standards and must be focused on students and their learning. It is necessary to improve the strategies to check how students make progress. This will allow making pedagogical decisions coherent to that progress, and it is one of the keys to include formative assessment in the teaching process. As mentioned, teachers have little time, and their evaluation training is limited. So, to have a tool easy to build (as to identify key concepts and include them in disjunctive tasks such as those described above) which does not explicitly ask about a particular content (which in turn could decrease the anxiety the traditional assessment causes) could support students' learning process. This would be an assessment focused on the student, and not very demanding for the teacher. It would enable to spend more time, for example, on an appropriate feedback. There is strong evidence regarding the need to give timely feedbacks in the learning process (see, e.g., Zepeda, 2023).

For instance, it is customary to teach a disciplinary content in a few classes. Then, there is an assessment. Teachers often have results next week. Thus, the time cycle is about three to four weeks. It is possible to check every class (e.g., with short questions about the learned contents), but that means additional work for teachers to review the answers. If there were tests simple to build, it would not be hard to check all the students and have results more promptly. All this would allow setting the next class and improve the learning cycle.

Therefore, if the theory of mental models is assumed and the experimental results supporting its account of the disjunction problem (e.g., in Orenes & Johnson-Laird, 2012) are considered, it is possible to build an assessment instrument based on the theses of the theory. The predictions indicated in the previous section are only valid if individuals know the concepts of dog, cat, and feline. The inference of (2) from (1) can only be rejected if it is known that a cat and a dog are different animals. To know that implies to know that if an animal is a cat, that animal is not a dog. This knowledge is what makes (2) incoherent with (1): individuals know that one possibility for (2) is that my pet is a cat and not a dog.

Nevertheless, the case of (17) and (19) is different. The knowledge that cats are felines is what makes the inference acceptable. If it is known that it is impossible that an animal is a cat without being a feline, it is not possible to think about a model derived from (17) incompatible with premise (19). Accordingly, an inference such as this one could be useful to check whether students know that cats are felines or not. If they know it, they will admit the inference of (17) from (19). If they do not know it, they will not admit that very inference.

Two more examples can illustrate this point. A frequent mistake is to think that tomato is a vegetable. It is actually a fruit. So, teachers could work this point with students and try to check whether students have understood by means of an inference such as (20).

(20) $\dfrac{\text{I am going to eat fruit}}{\text{Therefore, I am going to eat tomato or I am going to eat fruit}}$

A possible question for students could be whether (20) is correct or not. If the response is affirmative, they know that tomato is a fruit. If the answer is negative, they do not know that. This is because if students do not know that tomato is a fruit, they will attribute to the conclusion in (20) the possibilities in (21).

(21) Possible (I am going to eat tomato & I am going to eat fruit) & Possible (I am going to eat tomato & I am not going to eat fruit) & Possible (I am not going to eat tomato & I am going to eat fruit)

The second model in (21) would be inconsistent with the premise in (20). So, the inference would not be admitted.

Nonetheless, if students know that tomato is a fruit, the controversial possibility in (21) disappears. The set of models is transformed into (22).

(22) Possible (I am going to eat tomato & I am going to eat fruit) & Possible (I am not going to eat tomato & I am going to eat fruit)

The second possibility in (21) is not now. Students know that it is not possible that something is a tomato and is not a fruit at once. Therefore, no model is incoherent with the premise, and inference (20) should be accepted.

One more example can be linked to geography. Geography is a subject matter in which this kind of inferences can be relevant too. The acceptance or rejection of an inference such as (23) will show whether or not students know that Düsseldorf is in Germany.

(23) $\dfrac{\text{I am in Germany}}{\text{Therefore, either I am in Düsseldorf or I am in Germany}}$

According to what has been indicated, if students do not know that Düsseldorf is in Germany, they will consider a model in which I can be in that city without being in Germany. In particular, their models will be those in (24).

(24) Possible (I am in Düsseldorf & I am in Germany) & Possible (I am in Düsseldorf & I am not in Germany) & Possible (I am not in Düsseldorf & I am in Germany)

Again, the second possibility in (24) is in contradiction with the premise in (23). So, the inference would be rejected.

But if students know that Düsseldorf is a German city, this will not happen. The possibilities they will take into account will be just those pointed out in (25).

(25) Possible (I am in Düsseldorf & I am in Germany) & Possible (I am not in Düsseldorf & I am in Germany)

Conjunction of models (25) is not in conflict with the premise in (23). Therefore, the acceptance of (23) as a correct inference would reveal that students have the required geographic knowledge in this case.

2. Discussion and conclusions

The disjunction problem is that, from the logical point of view, it is valid to derive from a fact a disjunction including that fact as one of its disjuncts. However, this is something people do very rarely. The theory of mental models describes the situations in which they make this type of inferences. Those situations are the circumstances in which the possibilities or models of the disjunction are not incoherent with the fact the premise indicates.

The potential of this to create assessment instruments is huge. The inference related to (17) and (19), inference (20), and inference (23) show that. The knowledge and the lack of knowledge of certain concepts or definitions can lead to different results. Thereby, the proposal of the present paper refers to the use of inferences such as those analyzed above in assessment processes, whether the assessment is formative or summative.

The advantages would be several. On the one hand, the instruments would not be tasks hard to do. The disjunction problem addressed in this paper is often deemed as a 'paradoxical inference' (e.g., Orenes & Johnson-Laird, 2012), but it is not a paradox in the technical sense. If the theory of mental models is right, the tasks to do would be easy for students and they would give relevant information for teachers. The reasons why students can offer a wrong response in a task can be different. The instruments proposed in this paper can help detect what the exact reason of a mistake is. If instruments based on disjunction such as those described here are used together with other instruments, they could show, for example, that the latter instruments have to be reviewed. Correct responses in taks such as those indicated above and incorrect responses in other instruments evaluating the same concepts could be a signal that the latter instruments need to be improved.

What has been explained throughout this paper seems to indicate that there are already tasks based on the disjunction problem to check whether or not students have assimilated certain concepts. Teachers can verify whether students know the relation existing between two words when the extension of one of them is a subset of the extension of the other one. Extension is the number of elements a word refers to (e.g., Carnap, 1947). Accordingly, teachers only need to build inferences such as those addressed here. If the word with greater extension is included in the premise, it is possible to note whether students understand the relation. Examples such as those dealt with above (i.e., those of cat and feline, tomato and fruit, and Düsseldorf and Germany) support this idea.

Extension and subset relations are the key criteria. They allow creating assessment tasks in very different school subjects. This paper is not intended to present practical impletations of that criteria. It only shows the general lines to build tasks to check whether or not students know relations between concepts. There are examples in natural sciences (that of cat and feline, and that of tomato and fruit) and geography (that of Düsseldorf and Germany) above. An example for mathematics could be an inference such as "this number is a real number; therefore, either this number is a natural number or this number is a real number". If the inference is accepted, that will mean that the student knows that the real numbers include the natural numbers. If it is not accepted, that will mean that the student does not know that. Regarding an example for linguistics, the inference could be "this is written in a Romance language; therefore, either this is written in Portuguese or this is written in a Romance language". In this case, the acceptation will indicate that the student knows that Portuguese is a Romance language. The rejection will reveal that the student does not know that.

There are elements to continue to explore. One of them is the development of deductive reasoning. It is known that children make logical inferences from a young age. However, the development of a key function such as working memory can differentiate the access to correct conclusions. For example, the theory of mental models (Johnson-Laird & Byrne, 2002) predicts that young children can represent one initial model (related to conjunction), and older children can represent a double initial model (related to the biconditional).

One more point to take into account is the type of task. For instance, in evaluation tasks, the conclusion is presented from the beginning. On the other hand, in building tasks, the children should look for the conclusion. So, this is an issue about which there is a lot of work to do.

Besides, it has been suggested that, in the theory of mental models, disjunctive inferences can be different from inferences such as those transitive. Transitive inferences can be linear but disjunctive inferences likely involve multiple mutually exclusive models (Cesana-Arlotti & Kovács, 2020).

Perhaps, the most important limitation of this paper is that it is a theoretical proposal to work within assessment processes. It is necessary to check its real results in educational practice. Subsequent empirical studies might address this limitation. One more

limitation can be that many teachers do not know the theories about deductive and inductive reasoning. The development of the assessment instrument would be easy, but teachers should know the structure underlying logical reasoning. Accordingly, teachers would need a brief training. This proposal will only move forward with further consideration of these two limitations.

This is not the first time the theory of mental models is proposed in education. There are papers related to social beliefs and the conditional (e.g., Torres-Bravo & Gairín, 2019) and language learning (e.g., López-Astorga, 2019) following the theory as well. Likewise, the literature on the theory of mental models reveals that modulation can have an influence on more connectives, and not only on disjunction (for the particular case of the conditional, see, e.g., Orenes & Johnson-Laird, 2012). This means that it is also possible to think, from the theory of mental models, about other assessment instruments built from other connectives. Hence, in addition to use disjunctions such as those indicated in this paper in evaluation processes, it seems that further exploration about what the theory can offer to pedagogy is desirable.

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CRediT authorship contribution statement

Miguel López-Astorga: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Formal analysis, Conceptualization. **Leyla D. Torres-Bravo:** Visualization, Validation, Formal analysis, Conceptualization. **Cristian A. Rojas-Barahona:** Writing – review & editing, Writing – original draft, Visualization, Validation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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