Organon F 31 (1) 2024: 48–59
https://doi.org/10.31577/orgf.2024.31103
ISSN 2585-7150 (online)
ISSN 1335-0668 (print)

RESEARCH ARTICLE

Chrysippus' Conditional Captured from a Non-Axiomatic Computer Program

Miguel López-Astorga*

Received: 11 September 2023 / Accepted: 11 February 2024

Abstract: It is usually accepted that Chrysippus of Soli proposed the "connexivist view" of the conditional. I assume here that Chrysippus also supported the "inclusion view" and that differentiated between two kinds of conditionals: strong and weak conditionals. The latter assumptions allow me to link Chrysippus' interpretation of the conditional to a computer program such as Non-Axiomatic Reasoning System (NARS). The inclusion view enables to deem Stoic conditionals as inheritance relations in NARS. The distinction between strong and weak conditionals helps assign values of frequency and confidence such as those NARS inheritance relations have to Stoic conditionals.

Keywords: connexivist view; inclusion view; inheritance relations; non-axiomatic reasoning system; Stoic conditional.

1. Introduction

In the logic Chrysippus of Soli proposed, the conditional is not classical logic conditional. Chrysippus' view is often linked to the "connexivist" tra-

* University of Talca, Chile
https://orcid.org/0000-0002-6004-0587
Institute of Humanistic Studies, Research Center on Cognitive Sciences, University of Talca, Talca Campus, Chile
milopez@utalca.cl

dition (e.g., O'Toole and Jennings 2004). It has been addressed from different approaches, even from the modern framework of modal logic (e.g., Lenzen 2019). What I try to do here is to consider Chrysippus' interpretation from a current non-axiomatic system coming from Artificial Intelligence (AI). That system is NAL (Non-Axiomatic Logic) (Wang 2013).

The advantage of that consideration is twofold. On the one hand, because NAL is a term logic, it means to relate Stoic logic to a term logic, and, therefore, to a logic akin to the Aristotelian. On the other hand, given that NAL enables to build NARS (Non-Axiomatic Reasoning System), that is, a computer program (see also, e.g., Wang 2006), it also makes it possible to relate Stoic logic to AI. The latter relation is very interesting as NARS is based on an assumption (see also, e.g., Wang 2011): The Assumption of Insufficient Knowledge and Resources (AIKR). This assumption allows NARS to work in a cognitive context akin to that habitual for the human mind. Hence, to link the interpretation of the conditional Chrysippus of Soli gave to NAL can lead to think about Stoic logic as a logic near the manner human beings draw conclusions.

The paper will have three parts. First, I will discuss the way Chrysippus of Soli understood the conditional. To clarify this point is important to determine to what extent the relation to NAL is justified. Then, I will describe the characteristics and components of NAL necessary to link both approaches. The last part will indicate how the relation between Chrysippus' view of the conditional and NAL can be provided. It will also show how, by virtue of the relation, the Stoic conditional can be captured from a computer program such as NARS.

2. The conditional in Stoic logic

It can be thought that the way Chrysippus of Soli interpreted the conditional is the way the Old Stoa understood it (O'Toole and Jennings 2004). This way is different from that of classical logic. As indicated by Cicero (*Academica*) and Sextus Empiricus (*Adversus Mathematicos*, *Pyrrhoniae Hypotyposes*) there was an intense debate about the characteristics of the sound or true conditional in the fourth century B.C. It is not clear whether the words 'sound' and 'true' were synonymous in Stoic logic (for a discussion in the context of the debate, see, in addition to O'Toole and Jennings (2004), e.g., Mates (1953). However, because that does not have an influence on the develop of the present paper, I will assume that they were synonymous. Sextus Empiricus spoke about four different opinions in this way. One of them is that of Philo of Megara (Sextus Empiricus, *Adversus Mathematicos*, 8.113; *Pyrrhoniae Hypotyposes*, 2.110), which has been deemed as the account corresponding to modern classical logic and, accordingly, to the material view of the conditional (see also, e.g., Bocheński 1963). Nevertheless, that is none of the two accounts relevant here.

One of the accounts important for this paper is that claiming the need for a connection or bond between the two clauses, that is, the antecedent and the consequent. That connection is expressed as a fight between the opposite of the second clause, that is, the consequent, and the first clause, that is, the antecedent (Sextus Empiricus, *Pyrrhoniae Hypotyposes*, 2.111). Sextus Empiricus did not point out who the proponents of this account were. Nonetheless, following other ancient sources too (Cicero, *De Fato*, 12), most of the authors think that there is no doubt that this is Chrysippus' account (see also, e.g., Gould 1970). This view is often called the 'connexivist view' of the conditional (e.g., O'Toole and Jennings 2004).

While the connexivist view is hard to assume from propositional calculus, it is not from modern logic in general. For example, the relations to the strict implication (Lewis 1918) seem to be evident (for a discussion, see, e.g., Gould 1970); see also (Lenzen 2019), where links between the strict implication, the connexivist view, and Leibniz's work are provided). Nonetheless, to address one more interpretation of the conditional in the debate in the fourth century B.C. can be more interesting now. Other account Sextus describes is that of the philosophers considering the consequent to be included in the antecedent (Sextus Empiricus, *Pyrrhoniae Hypotyposes*, 2.112). This interpretation if often named the 'inclusion view' (e.g., (O'Toole and Jennings 2004)).

The inclusion view is relevant because it has been thought that it is not very different from the connexivist view (e.g., Kneale and Kneale 1962; Long and Sedley 1987). It has been said even that it is possible that Chrysippus offered the connexivist criterion in order to develop and clarify the inclusion criterion (O'Toole and Jennings 2004). Besides, the algebra

Leibniz proposed, which also presented 'containment relations' between two clauses, has been related to the connexivist view as well (Lenzen 2019).

All of this leads to think that there are reasons for assuming that the criterion Chrysippus of Soli supported includes both the connexivist view and the inclusion view. Thus, I will accept that assumption in this paper. To my aims here, only one more point needs to be reviewed. It seems that Chrysippus also distinguished between strong and weak conditionals (e.g., Sedley 1984). The thesis is based on sources such as Cicero (*Academica*, II.99-100; De *Fato*, 15-16). It appears to be that, when a conditional relation is coherent with Chrysippus' account, it should be worded in natural language as a conditional. But if we are not absolutely sure that the denial of the second clause is not compatible with what the first one states, it should be worded as a conjunction that is negated, and in which one of the conjuncts is negated too. The distinction Sedley supports is also addressed in works such as López-Astorga (2015a).

Let A be the antecedent in a conditional relation. Let B be its consequent. The idea in the previous paragraph means that, in a natural language such as English, if the conditional relation is clear (according to Chrysippus' view), the sentence indicating that relation should be "If A then B." If there is uncertainty on the conditional relation, the correct sentence should be "It is not the case that A and not B."

This thesis shows the distance existing between Stoic logic and modern propositional logic. In the latter, conditionals and negated conjunctions are interchangeable. Nevertheless, beyond the fact that the distance is obvious and has already been highlighted in different senses (e.g., Bobzien 1996), we have reasons for accepting the difference between strong and weak conditionals. Works such as that of Sedley (1984) illustrate the reasons. Therefore, I will assume the difference here as well. The difference will be essential for the relation I will establish between Stoic logic and NAL below.

3. Inheritance relations in NAL

NAL is a term logic (Wang 2013). It has several rules and a grammar so broad that it cannot be described in entirety in this paper. I will only refer to the components of NAL necessary to provide a relation to Chrysippus' logic.

In NAL, there subjects and predicates linked by means of inheritance relations. An inheritance relation between a subject 'S' and a predicate 'P' is expressed in NAL as (INH1).

(INH1) $\quad$ S → P

(INH1) literally appears and is explained in (Wang 2013, Definition 2.2).

The concepts of extension and intension define what inheritance relations are. All the subjects had by a predicate are the extension of the predicate. On the other hand, all the predicates had by a subject are the intension of the subject (Wang 2013, Definition 2.8). For example, let us suppose the following inheritance relations:

ARISTOTLE → PHILOSOPHER

CHRYSIPPUS → PHILOSOPHER

CICERO → PHILOSOPHER

ARISTOTLE → GREEK

ARISTOTLE → HUMAN BEING

If only these inheritance relations are considered (as a subset of all of the inheritance relations), it can be said that,

— "Aristotle," "Chrysippus," and "Cicero" are the extension of "philosopher."
— "Philosopher," "Greek," and "human being" are the intension of "Aristotle."
— "Philosopher" is the intension of "Chrysippus."
— "Philosopher" is the intension of "Cicero."
— "Aristotle" is the extension of "Greek."
— "Aristotle" is the extension of "human being."

But AIKR is essential in NAL. This is because we cannot be sure about the inheritance relations. For this reason, NAL inheritance relations have both a value of frequency "f" and a value of confidence "c." In this way, inheritance relations are not represented as (INH1), but as (INH2).

(INH2) $S \rightarrow P$ (f, c)

(INH2) appears and is explained in, for example, (Wang 2013, Definition 3.8).

The formulae to calculate f and c (as they are literally in (Wang 2013, Definition 3.3)) are:

Frequency: $f = W^{+}/W$
Confidence: $c = W/(W + K)$

Where "W^{+}" expresses "positive evidence," "W" is "total evidence," and "K" is a constant whose value is generally 1.

Although the definition of "W^{+}" is more complex (see Wang 2013, Definition 3.2), for the goals of the present paper, it is enough to assume that, given this inheritance relation,

PHILOSOPHER → GREEK

"Aristotle" is positive evidence, but "Cicero" is not. "Cicero" is part of the total evidence, but not positive evidence.

NAL includes different rules. However, to deal with the deduction rule as an example can suffice here. Following the schema of the latter rule in (Wang 2013, Chapter 4), this inference would be a case of deduction:

GREEK → PHILOSOPHER (0.8, 0.83)
LOGICIAN → GREEK (0.6, 0.91)

Therefore, LOGICIAN → PHILOSOPHER (0.48, 0.36)

The numbers in the three statements are explained as follows:

— We have checked five Greek people, and four of them are philosophers. Hence, $f = 4/5 = 0.8$; $c = 5/6 = 0.83$.
— We have checked ten logicians, and six of them are Greek. Hence, $f = 6/10 = 0.6$; $c = 10/11 = 0.91$.
— The frequency and confidence of the conclusion are calculated following the formulae indicated in (Wang 2013, Table 4.7). For frequency, the frequencies of the two premises are multiplicated. For confidence, four numbers are multiplicated: the frequencies and the confidences of the two premises. Hence; $f = 0.8 \text{ x } 0.6 = 0.48$; $c = 0.8 \text{ x } 0.83 \text{ x } 0.6 \text{ x } 0.91 = 0.36$.

Much more inference rules are in NAL. Nevertheless, this example of deduction is enough to present how the system works.

4. Stoic conditional and NAL

This is not the first paper trying to relate Stoic logic to the way human beings think. For example, there are works linking the latter logic to contemporary psychology theories such as the theory of mental models (e.g., Johnson-Laird 2023; Khemlani and Johnson-Laird 2022); an example of work linking Stoic logic to the theory of mental models is López-Astorga (2015b) and the mental logic theory (e.g., O'Brien 2014 and O'Brien 2021); an example of work linking Stoic logic to the mental logic theory is (López-Astorga 2015b). On the other hand, links between the connexivist view and a term logic are also to be found in the literature (e.g., Lenzen 2019). Nonetheless, as far as I know, there are not any works providing relations between Stoic logic and systems of AI such as NARS. I will establish a relation between Chrysippus' view of the conditional and NAL, which is the foundation of NARS, in this section.

To relate a Stoic conditional to a NAL inheritance relation is not difficult if it is assumed, as I did above, that Chrysippus' criterion is both the connexivist and the inclusion criteria. As indicated, there are reasons for the assumption. One of the reasons is particularly interesting. As also mentioned, Leibniz's algebra has been related to the connexivist view (Lenzen 2019). That algebra includes containment relations. In it, it is possible to state that "a term A contains a term B." In addition, the algebra also resorts to the concepts of extension and intension. So, one might think that the theoretical arguments to support that there are not many differences between the connexivist and the inclusion criteria are strong.

From this point of view, if the antecedent of a conditional includes the other clause, that means that the latter is part of the intension of the antecedent, and the antecedent is part of the extension of the second clause. Thus, if the antecedent of a conditional consistent with the inclusion account is deemed as "S" and its consequent is denominated "P," those clauses satisfy (INH1).

NAL inheritance relations are actually represented as (INH2). But this is not a problem if the distinction between strong and weak conditionals (e.g., Sedley 1984) is taken into account. If the frequency of an inheritance relation is not 1, that inheritance relation cannot correspond to a strong conditional. It can only capture a weak conditional, that is, a conditional about which we are not sure whether it is compatible with Chrysippus' interpretation or not. This is because an inheritance relation has value of frequency 1 only when all the cases reviewed are positive evidence, that is, are part of W^+. As far as confidence is concerned, it is obvious that it should be high as well. However, a value of confidence 0.9 is already achieved only checking ten cases ($c = 9/10 = 0.9$). This is important, since, given the formulae above, the value of confidence (and frequency) of any inheritance relation can only vary between 0 and 1.

Therefore, in an inheritance relation such as (INH2), if $f = 1$, then it should be formulated as "If S then P." If $f < 1$, then it should be couched as "It is not the case that S and not P."

As it can be noted, the computational processing of my proposal is really easy. Although the habitual languages for NARS at present are Prolog and Java (Wang 2013), I can offer a trivial example of code in Common Lisp (LispWork Personal Edition). The code is very simple, but it can differentiate between strong and weak conditionals by virtue of their values of frequency. It is the following:

```
(defun Chrysippus (L1 L2 N)
          (IF (= N 1) (append '(if) L1 '(then) L2)
                    (append '(it is not the case that) L1 '(and not) L2)))
```

In Common Lisp, "defun" allows defining functions. Here, the function created is named "Chrysippus." "L1," "L2," and "N" are the three variables of function "Chrysippus." "L1" corresponds to the first clause or antecedent. "L2" refers to the second clause or consequent. Variable "N" is the number for frequency. I resort to function "IF," which establishes a condition: N = 1 "(= N 1)." If that condition is correct, what is indicated between the next brackets, that is, "(append '(if) L1 '(then) L2)," is run. Function "appends" joins "if" + L1 + "then" + L2. If the initial condition is not correct, that is, $n \neq 1$, what is in the third line, that is, "(append '(it is not the case that)

L1 ‘(and not) L2)” is run. In the latter situation, “append” binds “it is not the case” + L1 + “and not” + L2.

If, for example, we write,

> Chrysippus ‘(dog) ‘(MAMMAL) 1

The system will return:

IF DOG THEN MAMMAL

If the information I give is,

> Chrysippus ‘(dog) ‘(dalmatian) 0.9

The answer will be,

IT IS NOT THE CASE THAT DOG AND NOT DALMATIAN

The first example corresponds to a conditional coherent with Chrysippus’ account. If an animal is not a mammal, that animal cannot be a dog. For this reason, the sentence is written as strong conditionals in Stoic logic. In the second example, there is no consistency with Chrysippus’ view. An animal may not be a dalmatian and keep being a dog. So, in the latter example, the formulation corresponding to weak conditionals in Stoic logic is that to be used.

The drafting of both function ‘Chrysippus’ and the information given in the examples can be improved. For instance, in the case of the information written, the data could be those:

> Chrysippus ‘(this is a dog) ‘(this is a mammal) 1

What the system would return would be,

IF THIS IS A DOG THEN THIS IS A MAMMAL

But the examples indicated make the point the present paper is intended to do.

Conclusions

It is possible to relate the logic Chrysippus of Soli proposed to a computer program making inferences in a similar manner as human beings.

Probably, there are many ways to do that. I have tried to do it resorting to NAL here.

We only need to assume two points. First, the connexivist and the inclusion criteria Sextus Empiricus distinguished are the same (or, at least, very alike) and, accordingly, both of them typify Chrysippus' account of the conditional. Second, Stoic logic differentiates between strong and weak conditionals. The first ones should be phrased as conditionals are habitually expressed. The second ones should be built by means of conjunctions, which in turn should be negated and have one of their clauses negated too.

This allows relating Stoic conditionals to NAL inheritance relations. The antecedent of a Stoic conditional can be deemed as the subject in a NAL inheritance relation. The consequent of that very conditional can be deemed as the predicate in that very inheritance relation. This is because, both in Stoic conditionals and in NAL inheritance relations, the first clause encloses the second one. The concepts of extension and intension enable to see this.

NAL inheritance relations have values of frequency and confidence associated. Hence, if an inheritance relation does not have the highest value of frequency (i.e., $f < 1$ in it), the Stoic conditional corresponding to it cannot be strong. Only the conditionals with $f = 1$, that is, with the highest value of frequency, can be considered strong conditionals in Stoic logic.

NAL is the logical skeleton of NARS. So, the arguments above allow linking Stoic logic to an AI program deriving conclusions in a similar manner as people do. Both Stoic logic and NARS include much more components than those addressed here. Nevertheless, to find a connection between the view of the conditional Chrysippus of Soli supported and NAL inheritance relations already shows that there is, at a minimum, one link between the two systems. Further research should try to relate more components from both frameworks.

Funding

PIA Ciencias Cognitivas, Centro de Investigación en Ciencias Cognitivas, Instituto de Estudios Humanísticos, Universidad de Talca.

Proyecto "Dialéctica virtuosa de la educación: De la fundamentación filosófica y antropológica a implementaciones concretas", código 15/I122 SAL127/19, UNMdP.

References

Bobzien, Susanne. 1996. "Stoic Syllogistic." In *Oxford Studies in Ancient Philosophy*, 133-192. Oxford: Clarendon Press.

Bocheński, Józef Maria. 1963. *Ancient Formal Logic*. Amsterdam: North-Holland.

Gould, Josiah B. 1970. *The Philosophy of Chrysippus*. Albany, New York: State University of New York Press.

Johnson-Laird, Philip N. 2023. "Possibilities and Human Reasoning." *Possibilities Studies & Society*, 1(1-2): 105-112. https://doi.org/10.1177/27538699231152731

Khemlani, Sangeet, and Philip N. Johnson-Laird. 2022. "Reasoning about properties: A computational theory." *Psychological Review*, 129(2): 289-312. https://doi.org/10.1037/rev0000240

Kneale, William and Kneale, Martha. 1962. *The Development of Logic*. Oxford: Oxford University Press.

Lenzen, Wolfgang. 2019. "Leibniz's Laws of Consistency and the Philosophical Foundations of Connexive Logic." *Logic and Logical Philosophy*, 28: 537-551. http://dx.doi.org/10.12775/LLP.2019.004

Lewis, Clarence Irving. 1918. *A Survey of Symbolic logic*. Berkeley: University of California Press.

Long, Anthony A., and Sedley, David N. 1987. *The Hellenistic Philosophers*. Cambridge: Cambridge University Press.

López-Astorga, Miguel. 2015a. "The Relationship between Conditionals and Denied Conjunctions in Stoic Logic." *Analele Universitatii din Craiova, Seria Filosofie* 36(2): 82-94.

López-Astorga, Miguel. 2015b. "Chrysippus' *indemonstrables* and mental logic." *Croatian Journal of Philosophy*, 15(43): 1-15. https://doi.org/10.5840/croatjphil20151511

Mates, Benson. 1953. *Stoic Logic*. Berkeley and Los Angeles: University of California Press. https://doi.org/10.1525/9780520349070

O'Brien, David P. 2014. "Conditionals and Disjunctions in Mental-Logic Theory: A Response to Liu and Chou (2012) and to López-Astorga (2013)." *Universum*, 29(2): 221-235. http://dx.doi.org/10.4067/S0718-23762014000200015

O'Brien, David P. 2021. "Natural Logic." In *The Handbook of Rationality*, 215-224. Cambridge, Massachusetts: The MIT Press. https://doi.org/10.7551/mitpress/11252.003.0021

O'Toole, Robert R., and Jennings, Raymond E. 2004. "The Megarians and the Stoics." In *Handbook of the History of Logic, Volume 1. Greek, Indian and Arabic Logic*, 397-522. Amsterdam: Elsevier. https://doi.org/10.1016/S1874-5857(04)80008-6

Sedley, David. 1984. "The Negated Conjunction in Stoicism." *Elenchos*, 5: 311-317.

Wang, Pei. 2006. *Rigid Flexibility: The Logic of Intelligence*. Dordrecht: Springer. https://doi.org/10.1007/1-4020-5045-3

Wang, Pei. 2011. "The Assumptions on Knowledge and Resources of Rationality." *International Journal of Machine Consciousness*, 3(1): 193-218. https://doi.org/10.1142/S1793843011000686

Wang, Pei. 2013. *Non-Axiomatic Logic: A Model of Intelligent Reasoning*. Singapore: World Scientific. https://doi.org/10.1142/8665