

MODULATION AND THE UNDERLYING ASSUMPTIONS OF THE PYTHAGOREAN BAN AGAINST EATING BROAD BEANS

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ABSTRACT. This paper tries to show that the theory of mental models describes deep mental processes that have to be assumed even by frameworks contrary to it. It has been argued that many explanations on certain cognitive activities different from that provided by the theory of mental models cannot ignore theses of this last approach. Those theses are related to the way the human mind interprets linguistic information and makes inferences. The main goal here is to give further evidence in this way by means of an analysis of a part of a fragment, authored by Diogenes Laërtius, about the Pythagorean ban against eating broad beans. The idea is to make it even more evident that any framework trying to account for how that part of the fragment can be understood by a reader needs to accept suppositions that characterize the theory of mental models.

KEYWORDS: general knowledge; information processing; mental models; modulation; Pythagoreanism.

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Introduction

An important theory of human reasoning is standing out nowadays. That theory is the theory of mental models (from now on, MM). Many relatively recent works explain its essential basis,¹ but the theory has been developing for decades.² If something seems to characterize MM, that is its idea that any sentence including some of the connectives that have been considered as traditional in classical logic (that is, mainly, the conditional [if... then...], conjunction [...and...], and disjunc-

¹ E.g., Byrne & Johnson-Laird (2020); Johnson-Laird, Quelhas, & Rasga (2021); Khemlani & Johnson-Laird (2022); Quelhas, Rasga, & Johnson-Laird (2017); Ragni & Johnson-Laird (2020).

² E.g., Oakhill & Garnham (1996).

tion [either... or...]) refers to a set of possibilities and, in particular, to ‘a conjunction of possibilities’.³ Those possibilities are obtained from the meanings of the words included in sentences, sometimes with the help of contexts and pragmatics as well. Such possibilities, which iconically describe alternatives of reality,⁴ are the elements leading reasoning: reasoning is just analyzing possibilities in order to draw coherent conclusions from them.⁵ So, this is an approach against the action of logic and logical forms in the human mind.⁶

Based on opposite perspectives, it has also been claimed that there is a logic in our mind, even if that logic is not exactly the classical one. An example in this regard can be the mental logic theory⁷, which appears to propose that we think resorting to logical forms akin to formulae and from which we derive logical conclusions by means of formal inference rules or schemata.⁸ In fact, following theories more or less similar to this one, it can be stated that even general knowledge can be related to formal structures. This in turn can allow explaining most of the inferences made by human beings in an essentially syntactic (with the meaning that this adjective has in cognitive science and linguistics) way, and, in addition, speaking about even a ‘syntax of thought’.⁹

Frameworks such as the last ones have supported their theses in different manners. One of them is the analysis of arguments offered by ancient thinkers.¹⁰ Nonetheless, this paper will try to show that what is correct is what is held from other perspectives¹¹: that MM is a better theoretical alternative referring to essential mental processes. On the one hand, those mental processes are necessary to identify the logical forms that theories such as that of the mental logic need. On the other hand, when those processes happen, the action of detecting logical forms is not necessary: it is possible to come to the same conclusions by virtue of just those processes and without the forms. This will be also shown here by means of the analysis, as an example, of an ancient philosophical fragment. In particular, the fragment that will be used will be one authored by Diogenes Laërtius about the Pythagorean prohibitions or abstinence rules. An argumentation on it will be provided. The characteristics of that argumentation will enable to

³ E.g. Khemlani, Hinterecker, & Johnson-Laird (2017).

⁴ E.g., Johnson-Laird, Khemlani, & Goodwin (2015).

⁵ E.g., Johnson-Laird (2012).

⁶ E.g., Johnson-Laird (2010).

⁷ E.g., O’Brien (2014).

⁸ E.g., Braine & O’Brien (1998a).

⁹ E.g., Braine & O’Brien (1998b).

¹⁰ E.g., López-Astorga (2017).

¹¹ E.g., Johnson-Laird (2010); López-Astorga (2016).

think that it can be easily applied to other fragments related to ancient philosophers (including those used in the literature to assert the mental logic theory) and everyday situations.

To work in that direction, the first section will explain what MM is exactly. The second one will present the Diogenes Laërtius' fragment mentioned and show how it seems to confirm the essential theses of MM. Then, the possible account that could be given from theories such as that of the mental logic will be also considered. Finally, it will be argued that this last account would depend on the mental processes described by MM. These last arguments will be useful to make it evident another relevant point indicated above: if the mental processes assumed by MM happen, logical forms and, therefore, the explanation that will be offered in the third section, become unnecessary and superfluous. Those forms are not a *conditio sine qua non* to explain the underlying reasoning of the fragment that will be reviewed. In fact, as suggested, if MM is accepted, the recovery of logical forms can be deemed as an additional effort that can be ignored without problems.¹²

MM and the mental activity

Maybe MM was presented as an alternative solution to the problem that people do not make or deem as suitable many logically correct inferences. MM proposes a framework that appears to be able to account for the human intellectual behavior, both in the cases in which the conclusions match classical logic and when they do not. An example can be enough to show this fact. That example will be linked to conditional reasoning.

According to MM, individuals tend to take into account the possibilities that can be derived from sentences. Thus, given a conditional such as 'if p, then q', people often think about three possibilities¹³:

[I] p & q

[II] Not-p & q

[III] Not-p & Not-q

¹² In this point, works such as that of Johnson-Laird (2010) or that of López-Astorga (2016) will be followed.

¹³ See, e.g., Johnson-Laird (2012, 138, Table 9.2).

[I], [II], [III] are not formulae, but iconic possibilities, iconic mental models, or, for the aims of this paper, even iconic possible worlds. This last expression, ‘possible worlds’, does not have here the same sense as it does in modal logic and works such as those of Kripke.¹⁴ The possible worlds in this paper are only descriptions of reality in an iconic way. The proponents of MM¹⁵ resort to works such as the one of Peirce¹⁶ to clarify to what extent those worlds are iconic.

Another important point is that, in this particular case, coincidentally, the three worlds match the situations in which a conditional is true in classical logic. Certainly, in this last logic, the conditional is only false in the case in which the first clause is true and the second one is false, that is, in the only case that is not included in the trio [I], [II], and [III]. This circumstance explains why the conclusions drawn by people are many times akin to those expected in classical logic. In many inferences, the models identified match the cases in which the connector (in the example here, the conditional) is true in logic.

Nonetheless, individuals do not always consider models such as [I], [II], and [III]. On the one hand, effort is sometimes necessary to do that. On the other hand, in other occasions, the meanings of the words and pragmatics modulate the possible scenarios and hence modify the possible worlds. These are the situations in which the conclusions given by individuals do not necessarily match those of classical logic. It is easy to check all of this if we pay attention to any logical rule that is controversial from the cognitive point of view. As the literature reveals¹⁷, one of those rules can be, for instance, the conditional introduction rule, that is, this rule:

$$[IV] q \therefore p \rightarrow q$$

(Where ‘ $\alpha \therefore \beta$ ’ means that β can be deduced from α , and ‘ \rightarrow ’ stands for the logical conditional relation).

This rule is controversial because people only sometimes apply or accept it. A formal theory more or less similar to the mental logic theory can explain the fact that individuals do not use it without difficulties. It is enough, for example, to claim that the real logic leading the human mind is a logic similar to the classical one but not identical to it. Thus, one of the differences between them is precisely that [IV] is not a valid rule in the true mental logic. Nevertheless, this may not

¹⁴ Kripke (1963a); Kripke (1963b); Kripke (1965).

¹⁵ See, e.g., Johnson-Lair et al. (2015).

¹⁶ Peirce (1931-1958).

¹⁷ See, e.g., Orenes & Johnson-Laird (2012).

solve the problem, since what has to be accounted for is not only why people do not apply that rule, but also why there are other situations in which it does be used.¹⁸

Against this, it can be said that the mental logic theory has offered an account of some limitations or restrictions that [IV] can have.¹⁹ But, beyond this discussion, what is really interesting for this paper is the explanation that MM provides in this regard. That explanation predicts the exact situations in which [IV] will be used and will not be used. According to it, it is evident what happens in the case of conditionals such as the following:

[V] “...If Lucas watched a movie then he didn’t watch the news...”²⁰

It is evident that [V] is a conditional (it includes the words ‘if’ and ‘then’) and that its models are as follows:

[I] (Lucas watched a movie) & (Lucas did not watch the news)

[II] (Lucas did not watch a movie) & (Lucas did not watch the news)

[III] (Lucas did not watch a movie) & (Lucas watched the news)

However, it can be expected that, in this case, given a premise such as $\neg q$ (where ‘ \neg ’ represents negation), people do not conclude $p \rightarrow \neg q$, that is, that, given the premise ‘Lucas did not watch the news’, people do not conclude [V]. The reason of this prediction is not difficult to understand. The problem is [III], as it contradicts what is indicated in the premise. [III] is a possible world in which Lucas watched the news and that possible world cannot be admitted at the same time as the premise, because it indicates that Lucas did not watch the news. So, beyond the principles and requirements of classical logic, it can be predicted that people do not will consider inferences such as that to be correct. Arguments of this type have been experimentally tested. The results have given a strong support to MM.²¹

A different case is that of a conditional such as this one:

¹⁸ See, e.g., Orenes & Johnson-Laird (2012).

¹⁹ E.g., Braine & O’Brien (1998c).

²⁰ Orenes & Johnson-Laird (2012, 375).

²¹ See, e.g., Orenes & Johnson-Laird (2012).

[VI] “...If Julio watched a movie then he didn’t watch *The Matrix*...”²²

The special characteristic of this kind of conditional is that it has only two models:

[I] (Julio watched a movie) & (Julio did not watch *The Matrix*)

[II] (Julio did not watch a movie) & (Julio did not watch *The Matrix*)

A world such as [III] is not possible now because the meaning of the words in [VI], that is, semantic (in the manner this last word is understood in linguistics and cognitive science) factors and hence general knowledge, modulates the possibilities and blocks it. In [III] Julio would not watch a movie and he would watch *The Matrix*. That cannot be possible. *The Matrix* is a movie.

Following MM, now, $p \rightarrow \neg q$ could be drawn from $\neg q$, that is, [VI] could be drawn from a premise such as ‘Julio did not watch *The Matrix*’. The reason is clear. The model that is inconsistent with the premise, that is, [III], does not exist here. Therefore, there is no incompatibility, [IV] can be applied, and the derivation can be made. This is a prediction that has been experimentally tested and confirmed, too.²³

Thus, according to the proponents of MM, these facts are proofs that reasoning is as they describe, and not as logic prescribes. Nonetheless, what is relevant for this paper is not just how MM accounts for conditional reasoning in particular, but also its general conception of iconic models. That conception can help understand how individuals process arguments such as that related to the Pythagorean bans that will be dealt with below. In this way, it seems to be necessary, before continuing, to comment on an additional point of the theory. In relatively recent works supporting it²⁴, sets such as the one of [I], [II], and [III], or such as the one of just [I] and [II] in the case of [VI], are deemed, as indicated above, as ‘conjunctions of possibilities’. This means that, taking those works into account, a better expression for the possible worlds of [V] can be:

[VII] **possible($p \ \& \ q$) & possible(*Not*- $p \ \& \ q$) & possible(*Not*- $p \ \& \ \textit{Not}- q)$**

And for those of [VI]:

²² Orenes & Johnson-Laird (2012, 375, italics in text).

²³ E.g., Orenes & Johnson-Laird (2012).

²⁴ E.g., Khemlani et al. (2017).

[VIII] **possible($p \ \& \ q$) & possible($Not\text{-}p \ \& \ q$)**

These will be the types of expression that will be used from now on to refer to the models corresponding to sentences, ideas, and inferences. Accordingly, those types will also be those that will be used below to show that the view of MM about possibility and iconicity in human thought can be very useful to interpret fragments such as that of Diogenes Laërtius mentioned above.

Iconic models and the Pythagorean rule forbidding eating broad beans

To use MM to review ancient arguments and to compare that analysis to what can be said from a formal theory has already been done in several works.²⁵ The study that will be presented below can be considered one more piece of evident in the same direction as those works, that is, supporting the idea that those ancient arguments can be better captured by MM than by logic. But the study here refers to the prohibitions of a system of thought, the Pythagorean one, which appears not to have been addressed from this perspective yet.

This paper will focus on one particular fragment of that system of thought.²⁶ What will be interesting here is its first sentence, which is the following²⁷:

φησὶ δ' Ἀριστοτέλης ἐν τῷ περὶ τῶν Πυθαγορείων παραγγέλλειν αὐτὸν ἀπέχεσθαι τῶν κυάμων ἥτοι ὅτι αἰδοίοις εἰσὶν ὁμοιοὶ ἢ ὅτι Ἄιδου πύλαις...

[In *On the Pythagoreans*, Aristotle states that (Pythagoras) provided to refrain from eating broad beans, the reason being either that they are akin to the private parts or they are akin to the Hades doors...]

Diogenes names more reasons for not eating broad beans given by Aristotle. However, the two indicated in the quote (that they are similar to the private parts and that they are similar to the Hades doors) can be enough to develop in this section the idea that MM is able to describe in detail the way the human mental processes happen. The main point in this regard is that there is information that is missing. The fragment says that broad beans (in text, τῶν κυάμων) are forbidden to eat (in text, ἀπέχεσθαι) because either, first reason, they are akin to the private parts (in text, αἰδοίοις) or, second reason, they are akin to the Hades doors (in text, Ἄιδου πύλαις). Nevertheless, it is not said why exactly the private parts or the

²⁵ E.g., López-Astorga (2016).

²⁶ Fragment 275 in Kirk and Raven (1987).

²⁷ Diogenes Laërtius is who is talking. The entire fragment is to be found in *Vitae Philosophorum VIII*, 34-5.

Hades doors lead to the ban. One might think that an underlying idea of the fragment is that the private parts and the Hades doors have to be rejected and avoided, and that the similarity between broad beans and them causes broad beans to have to be rejected and avoided, too. This in turn leads to the ban to eat them. But this is not what is said in the text reproduced. It only states that broad beans cannot be eaten because of their resemblance to the private parts or the Hades doors. Nevertheless, although this is the case, MM can account for the intellectual process linking the mentioned likeness of broad beans to the private parts and the Hades doors to the prohibition to eat broad beans.

It can be thought that, in the abstract and without paying attention to any context, the iconic models relating broad beans to the fact that they should or should not be eaten, the private parts, and the Hades doors can be possible worlds akin to the following:

[IX] **possible**[(*broad beans are similar to the private parts*) & (*broad beans can be eaten*)] & **possible** [(*broad beans are similar to the private parts*) & (*broad beans cannot be eaten*)]

[X] **possible**[(*broad beans are similar to the Hades doors*) & (*broad beans can be eaten*)] & **possible**[(*broad beans are similar to the Hades doors*) & (*broad beans cannot be eaten*)]

Given that it can be assumed that broad beans are akin to both the private parts and the Hades doors, in [IX] possible worlds in which broad beans are not similar to the private parts cannot be included, as well as in [X] possible worlds in which broad beans are not similar to the Hades doors cannot be included either. Nonetheless, the most important point here can be that, both in [IX] and in [X], a situation akin to that of [VI] indicated above happens. As explained, in [VI] the possibility [III] disappears and the result is a set such as [VIII]. In the same way, in the fragment cited at the beginning of this section, Diogenes Laërtius is approaching a reader with a specific general knowledge, which leads to consider, both in [IX] and in [X], the first conjunct to be impossible.

The reader to whom Diogenes Laërtius is speaking knows that what should be rejected should be avoided, too. Hence, if it is food, it should not be eaten. That reader also knows that what is similar to the private parts should be rejected, which means that the reader also knows that what is similar to the private parts should be equally avoided. Hence, if it is food, it should not be eaten. This removes the first conjunct in [IX], which is transformed into:

[XI] possible[(*broad beans are similar to the private parts*) & (*broad beans cannot be eaten*)]

But, if there is only a possibility, it refers to the only action that can be done. In fact, following MM, a particular clause “is deemed *factual* if it is affirmed in a set of only one possibility”.²⁸ So, in this specific case, it is evident that what is provided in [XI] becomes a norm to follow. It is not a possibility anymore and becomes the only situation that can be considered.

Something similar is what occurs with [X]. It is known that what is akin to the Hades doors should be rejected. So, it is known that what is akin to the Hades doors should be avoided. Therefore, if it is food, it should not be eaten. Thus, the first conjunct is also eliminated in [X], the result being:

[XII] possible[(*broad beans are similar to the Hades doors*) & (*broad beans cannot be eaten*)]

Again, only one possibility can be thought, which reveals that what is expressed in that possibility is the only alternative. This last alternative, because it is the only one, it is transformed into a rule to fulfill, too.

So, it is clear that the iconic possible worlds of MM allow describing the underlying mental processes of the understanding of ideas such as that of the fragment analyzed. The reasons why the similarity to the private parts and the Hades doors lead to the prohibition to eat broad beans are in individuals’ general knowledge. This general knowledge exerts their action in the process of construction of models of reality and causes the disappearance of impossible or not enabled situations. This reveals the path to follow.

Nevertheless, from the point of view of a formal theory proposing that human thinking is linked to a logical system more or less related to the classical one, an objection could be raised against all of this: the previous account could also be given by means of logical forms and inference schemata. The next section shows a way an objection of this kind could be presented.

The Pythagorean ban to eat broad beans and logical forms

The formal theories are different from each other. For example, the mental logic theory is not exactly the framework held in works such as those of Henlé²⁹ or

²⁸ Khemlani et al. (2017, 261).

²⁹ Henlé (1962).

Rips.³⁰ Nonetheless, maybe the differences between those theories are not relevant for what will be argued in this section. It can be thought that the explanation below could be shared, at least in general terms, by all of them. In this way, although some of the formal theories distance themselves from classical logic to a greater or lesser extent and do not necessarily admit its symbols, its rules, and its way to build well-formed formulae, in the explanation here classical first-order predicate logic symbols, rules, and well-formed formulae will be used. The reason for this is that it seems that it would not be hard to adapt an explanation with those elements to the particular framework of each of the formal theories.

A first step necessary to start with the explanation can be to provide these equivalences:

$F\alpha$: to be forbidden eating α .

$P\alpha$: α is akin to the private parts.

$D\alpha$: α is akin to the Hades doors.

b : broad beans.

Given these equivalences, it can be said that the part of the fragment reviewed in this paper³¹ provides, on the one hand,

[XIII] $Fb \wedge Pb$

(Where ' \wedge ' is conjunction).

That is, that it is forbidden eating broad beans and broad beans are akin to the private parts, and, on the other hand,

[XIV] $Fb \wedge Db$

That is, that it is forbidden eating broad beans and broad beans are akin to the Hades doors.

Nonetheless, the word ' $\zeta\tau\iota$ ' twice in the text, which can refer to 'cause' and, although those are not the exact translations chosen here, can be even translated as 'because' or 'since', suggests that the resemblance to the private parts and the Hades doors is the reason why broad beans cannot be eaten. Thus, pragmatics,

³⁰ Rips (1994).

³¹ Fragment 275 in Kirk and Raven (1987).

whose action is acknowledged by most of the formal theories,³² can lead to implicit premises such as:

$$[XV] \quad \forall x (Px \rightarrow Fx)$$

(Where '∀' is the universal quantifier).

$$[XVI] \quad \forall x (Dx \rightarrow Fx)$$

These two last formulae are interesting because, by elimination of the universal quantifier, from [XV] it can be deduced

$$[XVII] \quad Pb \rightarrow Fb$$

And from [XVI] it can be derived

$$[XVIII] \quad Db \rightarrow Fb$$

[XVII] and [XVIII] reveal the cause-and-effect relation between the prohibition to eat broad beans and their likeness to the private parts or the Hades doors. In fact, by eliminating the conjunction in [XIII] and via Modus Ponendo Ponens, Fb can be obtained from Pb and [XVII]. In the same way, by removing the conjunction in [XIV] and via Modus Ponendo Ponens, too, Fb can be drawn from Db and [XVIII].

However, a problem continues to exist. As in the previous section, something is missing: the reason why the fact that something is similar to the private parts or the Hades doors leads to the ban to eat it. A manner to solve this difficulty can be, for example, as done above, to consider individuals to know that, if something is akin to the private parts, then that should be rejected, that, if something should be rejected, then that should also be avoided, and that, if something should be avoided, if it is food, then that should not be eaten. So, from a formal perspective, it can be thought that general knowledge provides more implicit premises, which are related to these last ideas. To indicate which those premises could be, it is necessary to assume two more equivalences:

$R\alpha$: α should be rejected.

$A\alpha$: α should be avoided.

³² See, e.g., for the particular case of the mental logic theory, Braine & O'Brien (1998b).

Thus, the premises should be:

$$[\text{XIX}] \quad \forall x (Px \rightarrow Rx)$$

$$[\text{XX}] \quad \forall x (Rx \rightarrow Ax)$$

$$[\text{XXI}] \quad \forall x (Ax \rightarrow Fx)$$

From Pb, by removing the universal quantifiers and resorting to Modus Ponendo Ponens again, by means of [XIX], [XX], and [XXI], Fb can also be deduced.

In the same way, for the case of the similarity to the Hades doors, it can be supposed, in addition, that people know that, if something is akin to the Hades doors, then that should be rejected, that is, that

$$[\text{XXII}] \quad \forall x (Dx \rightarrow Rx)$$

As in the account for the previous resemblance, from Db, removing the universal quantifiers and resorting to Modus Ponendo Ponens once again, by means of [XXII], [XX], and [XXI], Fb can be deduced.

Therefore, it can be said that an explanation based mainly on logical forms and inference rules of the same cognitive process addressed in the previous section can also be given. However, one might think that the formal explanation depends on the iconic account based on MM. The next section develops this point.

Relations between iconic possible worlds and logical forms

The previous logical explanation would probably be accepted by most of the formal theories. The resource to implicit or hidden premises is relatively frequent in them.³³ Besides, that thesis about hidden premises has already been used in other works to account for, from a formal perspective, problematic arguments proposed by ancient thinkers.³⁴ In this way, the explanation, in addition to seem correct, appears to be consistent with the specialized literature, too.

Nevertheless, even if this were the case, the formal theories would still have important problems to solve. For example, the difficulties involved in tasks including sentences such as [V] and [VI] are more clearly explained by MM than by

³³ See, e.g., Johnson-Laird (2010).

³⁴ See, e.g., López-Astorga (2017).

most of the formal theories. As shown, MM indicates when a rule such as [IV] will be applied and when it will not. But, beyond that problem and others like it that have not been overcome by many formal theories yet,³⁵ the most important point for this paper is perhaps that it can be thought that the formal explanation in the previous section is only possible if the iconic one above is accepted before.

It has already been proposed that, to come to logical forms, it is necessary previously to consider iconic possible worlds such as those of MM.³⁶ Similar arguments can be presented here. To obtain formulae such as [XV] and [XVI] is only possible if the meaning of words such as ‘ὄτι’ is known and the role pragmatics plays in language is taken into account. Many formal theories assume that pragmatics can be essential in reasoning. Nonetheless, to consider both pragmatics and the meaning of the words in sentences at the same time is an activity closer to the modulation processes of MM than to the formal frameworks (which often favor formal structures over meanings). In addition, it is very probable that logical forms cannot be found without that modulation activity.³⁷

Semantics (again, as this word is understood in linguistics and cognitive science) and pragmatics are also necessary to identify formulae such as [XIX], [XX], [XXI], and [XXII]. To build them, it is required to know the meaning of verbs such as ‘reject’, ‘avoid’, ‘forbid’ and ‘eat’, as well as the contextual components that can lead to that certain elements or objects (broad beans, the Hades doors) have to be rejected, hence avoided, and, accordingly, cannot be eaten. Therefore, it is necessary to repeat the mental iconic processes that take one from [IX] to [XI] and from [X] to [XII], but with an additional and harder activity: to translate what is claimed in the steps of those processes into logical well-formed formulae.

But, if all of this is correct, to do what has been done in the previous section, it is required to do what has been done to transform [IX] into [XI] and [X] into [XII]. This in turn seems to imply to assume several points. One of them is that the human mind modulates possibilities. The second point is that the human mind eliminates scenarios considered inconsistent (e.g., in the case of the sentence analyzed here, scenarios in which something should be avoided and can be eaten at the same time). The third point is that, hence, the mental processes happen such as MM proposes. The last point is that logical forms do not emerge *ex nihilo* or in an automatic way from the literal expressions as they appear in natural language (which is what the formal theories many times seem to state). On

³⁵ See, e.g., Orenes & Johnson-Laird (2012).

³⁶ See, e.g., for the particular case of a sophism attributed to Chrysippus of Soli or Eubulides, López-Astorga (2016).

³⁷ E.g., López-Astorga (2016).

the contrary, as already indicated in the literature,³⁸ they can come from iconic scenarios human beings construct when interpreting those expressions.

However, the account based on MM by itself already explains the mental process that occurs when individuals read sentences. So, if the framework of MM already gives an account, why could a recovery of logical forms from possible worlds identified in the way indicated by MM be necessary? To respond to this question can be hard. After identifying iconic scenarios following mental processes such as those described by MM, that recovery can seem a useless task.³⁹ If an explanation is already had, the search for other alternative account that needs to assume what has already been assumed previously, too (and, therefore, implies a higher degree of complexity) does not seem to be justified. Given these facts, if *Lex Parsimoniae* is taken into account, it is absolutely evident which the better option is.

Conclusions

Furthermore, it would not be difficult to check that arguments akin to those of this paper could also be offered in other cases. For example, they could be used with fragments related to other thinkers or everyday expressions that seem to prove that the human thought works resorting to implicit formal premises (even if they have already been reviewed from formal frameworks, as the case of, e.g., the argument Deng Xi proposes with regard to the price of a body⁴⁰). As shown, it appears that, to detect implicit premises, it is necessary the previous construction of iconic models of reality revealing possibilities. However, as also pointed out, once this last activity is done, the search for missing premises and the recovery of their logical form can be a superfluous activity.

Following a number of works about these issues,⁴¹ another important point should be acknowledged. If, although that is not necessary to account for the intellectual activity, it is possible to find correspondences between the iconic possible worlds of MM and logical forms, it is also possible that human reasoning does not lack any kind of logic. Continuing to follow works such as the last ones that have been indicated, it is very possible that human beings think as MM proposes, but that fact does not imply that their conclusions are contrary to logic. In other words, the human mind can work considering iconic models and possibili-

³⁸ E.g., López-Astorga (2016).

³⁹ E.g., Johnson-Laird (2010); López-Astorga (2016).

⁴⁰ See López-Astorga (2017).

⁴¹ E.g., López-Astorga (2016).

ties built from the meanings of words and expressions, and pragmatics. Nevertheless, if, after that, it is possible to reconstruct a formal explanation based on such models and possibilities, it can be said that the conclusions of the human inferential activity are consistent with logic, too. Neither people follow logic nor they do all it allows them to do, but the results of reasoning never seem to be against the main logical principles.

As those works⁴² suggest as well, this is a theoretical path that deserves further exploration. This is because, among other reasons, maybe that exploration can lead to identify something mentioned above and that some formal theories, including the mental logic theory, have looked for: the syntax (again, as the word is used in linguistics and cognitive science) of thought.⁴³ If reasoning is as MM proposes, it can be thought that, since this last theory resorts to neither logical forms nor inference rules, that syntax does not exist. Nonetheless, if formal correspondences to iconic possibilities such as those provided in this paper can be found, it seems possible, at least somehow, that MM is compatible with a syntax of that type. The only nuance in this regard would be that the syntax of thought would not be as it is proposed by formal theories such as the mental logic theory. In any case, the results that research in this direction can achieve enable to think that tasks such as those carried out in this paper should continue. To study problems such as the one dealt with in the present paper can be not only interesting, but also necessary.

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⁴² E.g., López-Astorga (2016).

⁴³ E.g., Braine & O'Brien (1998b).

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