DISJUNCTION: ITS INTERPRETATIONS AND L-TRUTH

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Abstract.   It has been proposed that disjunction has seven possible interpretations. Thus, it has been claimed that, in addition to the exclusive and inclusive ones, there are five more types of disjunctions. This paper analyzes those seven kinds of disjunction under Carnap's approach. The goal is to find examples of L-true sentences corresponding to each of the interpretations. The methodology used to carry out the study is based on the theory of mental models. The results show that there can be L-true disjunctive sentences for every type of disjunction.

Keywords: Carnap • disjunction • interpretation • L-truth • mental model

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1. Introduction

Traditionally, in logic it is understood that disjunction can be only either exclusive or inclusive. The difference between them is easy to capture. When exclusive, its two disjuncts cannot be accepted at once. An example can be this one:

(1) Either you are walking or you are sitting.

This disjunction is true when one of the disjuncts ('you are walking' and 'you are sitting') is that. It is not true when none of them is true (e.g., when you are running or you are standing). However, (1) is not true either when the two disjuncts happen, since that situation describes an impossible scenario: nobody can be walking and sitting at the same time.

In the case of inclusive disjunction, nevertheless, this last circumstance changes. As in exclusive disjunction, it is false if none of the disjuncts occurs, and it is true when one of them is the case. But, unlike sentences such as (1), inclusive disjunction can be true even in contexts in which the two disjuncts are real. That is because, when the particular disjunction is inclusive, such contexts refer to possible situations. Here is an example in this way:

(2) You are watching television or you are eating popcorn.

This sentence is true both if you are watching television and if you are eating popcorn. It is false if you are neither watching television nor eating popcorn. Nonetheless, in (2), it is possible that the two disjuncts happen at the same time (you can watch television while you are eating popcorn). In that case, (2) would be true too (for a more detailed explanation of these two types of disjunctions and the manner human beings tend to understand them, see, e.g., Khemlani, Orenes and Johnson-Laird 2014).

This distinction between exclusive and inclusive disjunction has caused interesting debates. There are many reasons for that. One of them can be, for example, that modern propositional calculus often privileges inclusive disjunction, as that calculus deems disjunction in a similar way as it is interpreted in frameworks such as the one of Gentzen (1934; 1935). However, in ancient philosophical approaches such as Stoic logic, the disjunction is basically exclusive (see, e.g., Lukasiewicz 1967, or O'Toole and Jennings 2004). In addition, contemporary psychological researches seem to show that people usually consider disjunction as exclusive (e.g., Khemlani, Orenes and Johnson-Laird 2014).

Nevertheless, beyond those discussions, what is interesting for the present paper is that it has been claimed that those are not the only interpretations that a particular disjunction can have. Five more manners to understand it have been proposed, the total number of interpretations being seven (López-Astorga 2019a). What these seven kinds of disjunction are will be indicated below. But the aim here is to review them from another point of view.

As it is well known, Carnap (1947) developed the concept of 'analytic truth' coming from Kant (1998) and linked it to what in modal logic is usually denominated 'possible worlds' (actually, the real expression used by Carnap is 'state-descriptions', and not 'possible worlds'). Thus, he stated that a sentence is analytically true if there is no possible world accessible from the one in which the sentence is expressed in which that very sentence cannot be admitted as true. The real expression to which Carnap (1947) resorted to refer to that kind of sentence is 'L-true'. With it, he tried to indicate that the sentence is true because the definitions of the language being utilized provides that. This last one will be the expression that will be used from now on.

Thus, an interesting point one might raise is whether or not it is possible to build L-true sentences in the case of the seven interpretations that have been proposed for disjunction. As it is also well known, Carnap (1947) spoke about F-truth as well to denote something that can occur in some possible world but not in every possible world. That expression stood for factual truth and, therefore, if applied to sentences, it alluded to the situations in which they are true because of the facts, and not by virtue of the content relations that can exist between the words appearing in them. From this perspective, it can be thought that maybe the plasticity of language allows identifying seven interpretations, but only provided that the sentences linked to those

interpretations are F-true, that is, just describe circumstances in particular possible worlds that cannot be extrapolated to all the possible worlds. In fact, as it can be inferred from the account below, to find examples of F-true sentences for the seven interpretations of disjunction is not hard. Perhaps it is even trivial to indicate examples of that kind. For this reason, the aim of this paper will be the contrary, that is, to offer examples of L-true sentences for each of the seven mentioned interpretations. In this way, it will be tried to show that such interpretations can be used not only to represent exceptional cases, and that they can also establish relations between contents that can be accepted in all of the possible worlds.

But several points in this regard need to be previously clarified. First, although the method of extension and intension Carnap (1947) describes seems to be essentially semantic, an argumentation such as that below cannot ignore pragmatic components and aspects. So, as it can be checked in the next sections, the relations between the contents of the clauses of the examples will not be only semantic. In many cases, pragmatic links will be taken into account, too. Second, as it is well known, Carnap's (1947) method was proposed to work with well-regulated languages whose definitions are accurate. However, the present paper will focus on natural language. This is one more step forward from Carnap's (1947) goals. That step is possible because it is not hard to suppose that, in the examples that will be presented, the definitions of the words used are understood just in the sense indicated, with no room for ambiguities. Third, beyond the fact that the title of the book in which Carnap (1947) presents the method includes the words 'meaning' and 'semantics', in that book it is explicitly stated that certain L-concepts may be applied to predicates, relations, properties, sentences, propositions, etc:

> It is often convenient to apply the *term 'equivalent'*, and perhaps also the term 'L-equivalent', not only to designators but likewise *to the intensions* of those designators; thus not only to predicators (e.g., 'the predicators 'H' and 'F • B' are equivalent in S_1') but also to properties and relations (e.g.,'the property Human and the property Featherless Biped are equivalent'); and analogously not only to sentences but also to propositions. (Carnap 1947, p.24; italics in text)

In addition, while the process of detection of possibilities in the theory of mental models (which will be described and illustrated below) requires the action of semantics and pragmatics (i.e., the consideration of both meaning and contextual situations), once the possibilities are detected, they can reveal deep logical structures (e.g., López-Astorga 2020). Finally, Carnap (1947) understands the intension of a sentence as the proposition it transmits. This reference to meaning can justify even in a greater extent the link to the theory of mental models. The accounts below show this, and it is not a problem within Carnap's framework:

> The context always makes clear whether the term 'equivalent' is meant in the original or in the transferred sense; the former is the case whenever the term is applied to expressions in a language system, the latter whenever it is applied to intensions, hence to extralinguistic entities. (Carnap 1947, p.24)

Having mentioned this, first, what the seven interpretations are and how their existence can be argued will be explained. Second, seven examples of L-true disjunctions with the characteristics of, respectively, each of the seven interpretations will be indicated.

2. Disjunction and its seven interpretations

As mentioned, the idea that disjunction can have seven interpretations has been proposed (in particular, in López-Astorga 2019a). The idea is based upon the general theses of the theory of mental models (e.g., Byrne and Johnson-Laird 2020). According to this theory, disjunctive sentences (and conditional sentences too) describe 'conjunctions of possibilities' (e.g., Johnson-Laird and Ragni 2019). This means that, given (1), its conjunction of possibilities would be as follows:

(3) Possible (you are walking & you are not sitting) & Possible (you are not walking and you are sitting).

And that of (2) would be akin to this one:

(4) Possible (you are watching television & you are eating popcorn) & Possible (you are watching television & you are not eating popcorn) & Possible (you are not watching television & you are eating popcorn).

The theory of mental models presents in detail the psychological processes and the circumstances that can lead an individual to identify conjunctions of possibilities such as (3) and (4) (see also, in addition to Johnson-Laird and Ragni 2019, e.g., Johnson-Laird 2012; Khemlani, Hinterecker and Johnson-Laird 2017). However, an important element of the theory for this paper is that, although one might think that they do, conjunctions of possibilities such as (3) and (4) do not stand for logical truth tables. This is for several reasons. Some of them can be relevant here. First, if a row in a truth table represents the true case, the other rows have to stand for false cases. The rows cannot be true at once. Nevertheless, the possibilities in a conjunction of possibilities such as described by the theory of mental models are conjuncts. Hence, they have to be true at once (see, e.g., Johnson-Laird and Ragni 2019). In addition, the possibilities are not formulae. They iconically represent alternatives of reality, as far as working memory can do that (see, e.g., Khemlani, Byrne and Johnson-Laird 2018).

Finally, context and the meaning of the contents in sentences can modulate conjunctions of possibilities (e.g., Quelhas and Johnson-Laird 2017; Quelhas, Johnson-Laird and Juhos 2010). So, semantics and pragmatics play a decisive role in the theory (and, as indicated, both semantics and pragmatics will be considered below). (5) is an example showing this last aspect.

(5) "Lucia wore the bracelet or she wore jewelry" (Orenes and Johnson-Laird 2012, p.363, see also, e.g., López-Astorga 2019a, p.53).

As it can be derived from the account by Orenes and Johnson-Laird (2012), which is also commented on in papers such as that of López-Astorga (2019a), the conjunction of possibilities of (5) can only have two conjuncts:

(6) Possible (Lucia wore the bracelet & Lucia wore jewelry) & Possible (Lucia did not wear the bracelet & Lucia wore jewelry).

In (6), a conjunct such as the second one in (4) is missing. The reason is obvious. In the case of (5), the situation described by the scenario in which the right disjunct is true and the left one is not is an impossible situation, since one wears jewelry provided that one wears a bracelet.

Thus, as pointed out, based on ideas such as these ones, López-Astorga (2019a) has offered until seven different interpretations of disjunction, three with structures such as, respectively, the ones of (3), (4), and (6), and four more interpretations. Those interpretations (which are distinguished by López-Astorga, 2019a, with a purpose very different from the goal of this paper) are the following:

A.- Exclusive disjunction. Taking (3) into account, its conjunction of possibilities is:

(7) Possible (first disjunct & not-second disjunct) & Possible (not-first disjunct & second disjunct).

B.- Inclusive disjunction. Taking (4) into account, its conjunction of possibilities is:

(8) Possible (first disjunct & second disjunct) & Possible (first disjunct & not-second disjunct) & Possible (not-first disjunct & second disjunct).

C.- Disjunction with its second disjunct secured. Taking (6) into account, its conjunction of possibilities is:

(9) Possible (first disjunct & second disjunct) & Possible (not-first disjunct & second disjunct).

D.- Disjunction with its first disjunct secured. Its conjunction of possibilities is:

(10) Possible (first disjunct & second disjunct) & Possible (first disjunct & not-second disjunct).

E.- Disjunction which is a conjunction. Its conjunction of possibilities is:

(11) Possible (first disjunct & second disjunct).

F.- Disjunction which is a conjunction with its second conjunct negated. Its conjunction of possibilities is:

(12) Possible (first disjunct & not-second disjunct).

G.- Disjunction which is a conjunction with its first conjunct negated. Its conjunction of possibilities is:

(13) Possible (not-first disjunct & second disjunct).

One might think that this account just refers to particular instances in which disjunction can be true in accordance with its truth table in classical logic. Nonetheless, as mentioned, the account is outside the context of that logic and its truth tables. The possibilities are linked by conjunctions, iconically stand for reality, and can be modulated.

As also said, it is not difficult to imagine examples for A, B, C, D, E, F, and G that are F-true. However, the aim here is different: to look for L-true examples. Actually, there are several works linking the theory of mental models to Carnap's (1947) framework (e.g., López-Astorga 2020). However, in general, the discussion in those works is very far from the one here. For the present paper, the most important aspects are semantics and pragmatics, in the sense these words have in linguistics. From this perspective, the next sections are devoted to, respectively, the seven interpretations mentioned.

3. Exclusive disjunction

The cases in which a sentence cannot be true can also be a conjunction. At least, it seems that this is what can be derived from papers such as the one of Johnson-Laird and Ragni (2019). Nevertheless, it appears that, in that case, the conjunction would have to be of impossibilities. In this way, following (7), the conjunction of impossibilities of A would be:

(14) Impossible (first disjunct & second disjunct) & Impossible (not-first disjunct & not-second disjunct).

Thus, the task to carry out here would be to find an example with thematic content in which the conjuncts in (14) are really impossible by virtue of language, that is, for which there are no possible worlds in which such conjuncts can be the case. An example of that kind could be this one:

(15) Either you were born in this place or you were born in another place.

Obviously, the conjunction of possibilities of (15) has the same structure as that of (3) and (7), and is:

(16) Possible (you were born in this place & you were not born in another place) & Possible (you were not born in this place & you were born in another place).

And, from (14), it is possible to derive its impossibilities:

(17) Impossible (you were born in this place & you were born in another place) & Impossible (you were not born in this place & you were not born in another place).

But, if it is assumed that 'place' only refers to a particular physical location, (17) shows situations that are actually impossible. There is no possible world in which you were born in this place and you were born in another place at once. On the other hand, there is no possible world in which you were not born in this place and you were not born in another place either, since everybody has to born in some place. Therefore, it can be stated that (15) is L-true.

4. Inclusive disjunction

In this case, (8) reveals that there is just one impossibility:

(18) Impossible (not-first disjunct & not-second disjunct).

Nonetheless, to propose an example for (18) is not hard either:

(19) Either your last book is in this language or it is in another language.

Now, evidently, the structure is akin to the one of (4) and (8):

(20) Possible (your last book is in this language & your last book is in another language) & Possible (your last book is in this language & your last book is not in another language) & Possible (your last book is not in this language & your last book is in another language).

The first conjunct in (20) can be admitted because, as it is well known, there are, for instance, bilingual editions and books in several languages. Hence, the only impossibility for (19) should be, as indicated by (18):

(21) Impossible (your last book is not in this language & your last book is not in another language).

And (21) leads to think that there is no doubt that (19) is L-true. A book needs to be in some language. Accordingly, the scenario presented in (21) can never happen.

A possible objection in this regard could be that one might imagine some world in which, for example, books are not in languages, but made by means of another system. Nonetheless, following Carnap (1947), L-true sentences are L-true because the language used determines that. Thus, if it is accepted that the words 'book' and 'language' are interpreted as usual, (19) is L-true because those words determine that. Possible worlds would refer just to worlds to which the speaker and the listener can access from the world in which the sentence, in this case, (19), is expressed and deemed as true. Therefore, in those accessible possible worlds, definitions such as the one of 'book' and the one of 'language' would keep being understood as usual, and it would not be possible to think about situations in which, for example, books have characteristics other than those that books have in the initial possible world.

5. Disjunction with its second disjunct secured

Paying attention to (9), it is not hard to note that the conjunction of impossibilities for this interpretation have these two conjuncts:

(22) Impossible (first disjunct & not-second disjunct) & Impossible (not-first conjunct & not-second conjunct).

So, an example could be here:

(23) Either you live in Asia or you live in this solar system.

Indeed, the possibilities of (23) match those of (6) and (9):

(24) Possible (you live in Asia & you live in this solar system) & Possible (you do not live in Asia & you live in this solar system).

The first conjunct is possible because Asia is in this solar system and, hence, you can be in the two places at the same time. Likewise, the second one is also possible because this solar system is wider than Asia. Accordingly, someone can be in this solar system and not in Asia.

As far as the particular conjunction of impossibilities for (23) is concerned, following (22), it is:

(25) Impossible (you live in Asia & you do not live in this solar system) & Impossible (you do not live in Asia & you do not live in this solar system).

Again, the impossibilities presented in (25) are real impossibilities. The first conjunct is impossible because nobody can be in Asia without being in this solar system. The second one cannot be admitted because, at least under the current technological conditions, nobody can live, for the time being, out of this solar system. Those impossibilities show that (23) is L-true.

6. Disjunction with its first disjunct secured

The previous and this interpretation are closely connected. Thereby, to offer an account of 'disjunction with its first disjunct secured' can be enough to do the same as in López-Astorga (2019a), that is, to modify only certain key elements related to 'disjunction with its second disjunct secured'. In this way, based upon (22), it can be claimed that the impossibilities are:

(26) Impossible (not-first disjunct & second disjunct) & Impossible (not-first disjunct & not-second disjunct).

By doing so, to give an example, it can be sufficient to reverse the order of the disjuncts in (23):

(27) Either you live in this solar system or you live in Asia.

Which leads to the following conjunction of possibilities:

(28) Possible (you live in this solar system & you live in Asia) & Possible (you live in this solar system & you do not live in Asia).

The reasons why (28) is the conjunction corresponding to (27) are the same as the reasons why (24) is the conjunction corresponding to (23). Like that, the reasons why (25) describes the impossibilities of (23) are also the same as the reasons why (29) describes the impossibilities of (27).

(29) Impossible (you do not live in this solar system & you live in Asia) & Impossible (you do not live in this solar system & you do not live in Asia).

Nonetheless, if these last reasons are the same, it is clear that (29) refers to true impossibilities and that, hence, (27) is L-true as well.

7. Disjunction which is a conjunction

Conjunctions have three cases in which they are wrong. Taking (11) into account, that leads to this conjunction of impossibilities for 'disjunction which is a conjunction':

(30) Impossible (first disjunct & not-second disjunct) & Impossible (not-first disjunct & second disjunct) & Impossible (not-first disjunct & not-second disjunct).

However, this circumstance does not mean that to propose an example is harder in this case.

(31) Either I am a human being or I am a rational animal.

It is obvious that (31) only allows one possibility:

(32) Possible (I am a human being & I am a rational animal).

Human beings are rational animals and rational animals are human beings. Therefore, it can be understood that both concepts are linked. Because of that, the impossibilities would be those matching (30).

(33) Impossible (I am a human being & I am not a rational animal) & Impossible (I am not a human being & I am a rational animal) & Impossible (I am not a human being & I am not a rational animal).

The first conjunct in (33) is not possible because it stands for a being that is a human being but not a rational animal. The second one is not possible because, at least as far as scientific knowledge informs today and the word 'rational' is understood in current cognitive science theories such as the theory of mental models, there are no rational animals not being human beings. The third one is not possible because, given the definitions of 'human being' and 'rational animal', it is necessary for the speaker (in this case, me) to be both of them. Hence, once again, (31) is a L-true sentence.

8. Disjunction which is a conjunction with its second conjunct negated

From what has been explained above, and especially (12), it can be inferred that the impossibilities of this interpretation are captured by the following conjunction with three conjuncts:

(34) Impossible (first disjunct & second disjunct) & Impossible (not-first disjunct & second disjunct) & Impossible (not-first disjunct & not-second disjunct).

Despite the number of conjuncts in (34), to find a L-true sentence is not complex here either.

(35) Either I am a human being or I am a machine.

Evidently, (35) only can be related to one possibility:

(36) Possible (I am a human being & I am not a machine).

The reasons for (36) are simple. Irrespective of philosophies such as the Cartesian one, 'human being' and 'machine' are usually incompatible concepts, and I am speaking and I am a human being. So, the set of impossibilities could be:

(37) Impossible (I am a human being & I am a machine) & Impossible (I am not a human being & I am a machine) & Impossible (I am not a human being & I am not a machine).

The first scenario in (37) is impossible because, as stated, 'human being' and 'machine' are not compatible. The second situation is not possible because, as also said, I am who is speaking, and I am a human being. The reason for the third case is this last one too. Accordingly, an instance of L-true sentence can also be given for this interpretation.

9. Disjunction which is a conjunction with its first conjunct negated

In a similar manner as in López-Astorga (2019a), the way to deal with this last interpretation could be akin to that used with 'disjunction with its second disjunct secured' and 'disjunction with its first disjunct secured'. Nevertheless, obviously, the reference point would be here the previous section, that is, the section addressing 'disjunction which is a conjunction with its second conjunct negated'. Thus, it would only be necessary to change some elements of this last account. Based upon (13) and (34), the impossibilities would be:

(38) Impossible (first disjunct & second disjunct) & Impossible (first disjunct & not-second disjunct) & Impossible (not-first disjunct & not-second disjunct).

Thereby, to give un example, it would be only required, as in the cases of (23) and (27), to modify the order of the disjuncts in (35).

(39) Either I am a machine or I am a human being.

For the same reasons relating (35) and (36), the only possibility of (39) would be:

(40) Possible (I am not a machine & I am a human being).

And (38) and the explanation above for (37) would lead to this set of impossibilities:

(41) Impossible (I am a machine & I am a human being) & Impossible (I am a
      machine & I am not a human being) & Impossible (I am not a machine & I am
      not a human being).

Nonetheless, if the reasons and explanations that could be taken are the same, undoubtedly, (39) is L-true too.

10. Conclusions

Accordingly, it is possible to find cases of L-true sentences, in the sense indicated in the introduction of the present paper, for the seven interpretations of disjunction proposed by López-Astorga (2019a). Sentences (15), (19), (23), (27), (31), (35), and (39) are not examples making little sense. It is not difficult to think about pragmatic situations or contexts in which their use would be suitable. Focusing just on two of them for illustrative purposes, it can be thought, for example, about a situation in which a person performs a difficult intellectual task and another person claims that a task of that type could only be performed by a machine. In this context, it would not be rare that the first person responded something similar to what is indicated in (35) or (39).

Examples of F-true sentences for those very interpretations would also be easy to propose. Given what has been shown, to offer examples of that kind could be even a trivial task. And this beyond the fact that many of those examples are already to be found in the philosophy, linguistics, or cognitive science literature (e.g., López-Astorga 2019a).

However, regardless of all this, the results achieved in the present paper are relevant, at a minimum, in two senses. On the one hand, they reveal that the seven interpretations for disjunction described deserve further exploration. Because it is possible even to think about L-true sentences for them, it could be interesting to try to consider them as independent linguistic operators. Traditionally, disjunction can only be inclusive or exclusive. However, perhaps the arguments above authorize to at least analyze the possibility of elaborating a new theory of disjunction.

On the other hand, the results also reflect the validity that methods such as the one of Carnap (1947), that is, his method of extension and intension (or at least parts and updated interpretations of such methods), can keep having nowadays. Because of Carnap's method, and in order to strengthen the proposal to take the seven interpretations of disjunction into account, the search for L-true examples for them has

been raised in this paper. Furthermore, it can be added, in this way, that the idea of continuing to use some aspects of the framework by Carnap (1947) has already been presented in the literature (e.g., López-Astorga 2019b, 2020). So, this paper can be deemed as one more work insisting in that very idea.

Hence, the research areas to which the previous account refers are two. One of them points out the interest that keeping expounding on the different interpretations of disjunction can have. The other one shows how much fruitful can be to continue counting on approaches such as that of Carnap (1947), at a minimum, as reference frameworks. Maybe to work on these two areas will indicate to what extent that can enable to come to relevant conclusions in fields such as linguistics, philosophy, and psychology.

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