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IN FAVOR OF LOGICAL FORM

Abstract. This paper is intended to address the work "Against logical form", authored by Johnson-Laird in 2010. In it, based on the theory of mental models, Johnson-Laird claims that the way people interpret sentences in natural language has nothing to do with logic. This is because that action is not related to logical forms. According to him, the mental activity is mainly linked to semantics and pragmatics. However, here, following arguments provided by López-Astorga, I try to show that the theory of mental models can be linked to syntactic structures. My main point is made by dealing with an argument given in the mentioned Johnson-Laird's paper. In principle, that argument seems to undermine proposals such as that of López-Astorga. However, the present work proposes otherwise.

Keywords: logical form, mental models, pragmatics, semantics, syntax.

Introduction

Today there is a cognitive theory rejecting the thesis that human thinking is about logical forms and syntax (in the sense given to this last word in current cognitive science). That theory is the theory of mental models. Actually, there are two versions of the theory: the original version (e.g., Johnson-Laird & Byrne, 1991) and the updated one (e.g., Johnson-Laird, Khemlani, & Goodwin, 2015a; Quelhas, Rasga, & Johnson-Laird, 2017). In any case, and irrespective of the version, one of the essential ideas of this framework is that reasoning is iconic. Besides, it focuses on possibilities provided by semantics and, in some occasions, adapted or qualified by pragmatics (again, in the sense that those words have in contemporary cognitive science).

This last point is important because, even in critical works acknowledging the role of syntax such as the one of López-Astorga (2017a), it is not questioned. Indeed, López-Astorga (2017a) tries to prove that the theory of mental models can be related to syntax, and that, in fact, a methodology to

recover logical forms can be deduced from it. However, he does not object its essential hypothesis that mental information processing is mainly semantic (and pragmatic) and begins with semantics (and pragmatics).

In this way, the basic aim of this paper is to continue to give further arguments in the direction assumed in works such as that of López-Astorga (2017a). In particular, what I will attempt to do here is, also accepting the priority role of semantics (and pragmatics) in the human mind, to deal with one of the explanations offered in a former paper supporting the theory of mental models. That paper is "Against logical form", written by Johnson-Laird (2010), and the explanation appears to be clearly incompatible with proposals such as the one of López-Astorga. Thus, the goal is to show, once again, that, even considering the core theses of the theory to be correct, such theses do not have to lead to ignore syntax; they can be consistent with certain relations between logical forms.

To achieve that goal, the first section will be devoted to a general description of the way the theory of mental models works in general (I will also point out some points that are different in the two versions). Second, López-Astorga's (2017a) approach will be accounted for and it will be shown how that approach links the theory to logical forms. Finally, the explanation given by Johnson-Laird (2010) mentioned above will be addressed. That will allow showing that, even assuming to a large extent the theses of theory of mental models about semantics and pragmatics, that explanation has no an influence on López-Astorga's proposal.

The theory of mental models, possible scenarios, and the rejection of logical form

There are many works about the original version of the theory of mental models (see also, e.g., Johnson-Laird, Girotto, & Legrenzi, 1999; Oakhill & Garnham, 1996). I will refer to this version below with the abbreviation 'MM'. On the other hand, there are also a lot of production about the updated version (see also, e.g., Khemlani, Byrne, & Johnson-Laird, 2018; Ragni & Johnson-Laird, 2020). From now on, this second version will be called 'UMM'. In both cases, the theory analyzes almost all of the aspects involved in human thinking, including the role of the traditional logical connectives in reasoning (see, e.g., Johnson-Laird, 2012). Nevertheless, to know how MM and UMM truly work and why, in principle, seem to be incoherent with logical forms, it can be enough to consider, as an example, the conditional.

If human reasoning followed classical logic, it would have to be always possible to apply a rule such as Modus Tollendo Tollens. This rule consists of two premises: a conditional and the negation of its consequent. Thus, the conclusion is the negation of the antecedent of the conditional. Therefore, if we took the requirements of that logic into account in a natural way, it would have to be always possible to deduce [III] from [I] and [II].

[I] $p \rightarrow q$

(Where '$\rightarrow$' stands for conditional relation).

[II] $\neg q$

(Where '$\neg$' represents negation).

[III] $\neg p$

The reason for this is clear if we pay attention to what is provided by the truth tables of propositional logic. Using symbols akin to those utilized in works such as the one of López-Astorga (2017a), it can be said that there is an equivalence between [I] and [IV].

[IV] $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

(Where '$\wedge$' is conjunction and '$\vee$' is disjunction)

That equivalence is given by their truth-values, since,

[V] $v(\mathrm{I}) = v(\mathrm{IV})$

(Where 'v(n)' refers to the truth-value of n)

So, given [V], which is a version of formula (1) in López-Astorga (2017a), as it is well known and can also be inferred from the account presented in this last chapter, [I] can only be true if and only if [IV] is true too, This in turn means that there is only one case in which [I] can be false, that is, the case in which this formula is true:

[VI] $p \wedge \neg q$

[VI] is the only missing alternative conjunction in [IV]. Hence, if correct, what this shows is that, if [I] is true -and hence [IV] is true as well-, and [II] is also true, the only possibility is that [III] is true too. The reason for this

is that, in [IV], the only disjunct in which [II] is true is a disjunct in which [III] is true as well.

Many times, the accounts akin to the previous one are right and Modus Tollendo Tollens can be applied to a conditional and the negation of its consequent. However, this is not always the case. There are circumstances in which it cannot be done. Johnson-Laird (2010) comments on one of those circumstances. It is the one of this conditional:

> If she played a musical instrument then she didn't play a flute.
> She played a flute.
> So, she didn't play a musical instrument (Johnson-Laird, 2010, p. 201).

This is a problematic inference. Given that the logical form of the first premise ('If she played a musical instrument then she didn't play a flute') is:

[VII] $p \rightarrow \neg q$

And that the one of the second premise ('She played a flute') is:

[VIII] q

It is obvious that, just following simple descriptions of classical propositional calculus in introductory books (e.g., Deaño, 1999; Fisher, 2008; Restall, 2006), the conclusion ('she didn't play a musical instrument') can be soundly derived. This is because its logical form corresponds to [III].

Nevertheless, as indicated by Johnson-Laird, people are unlikely to make an inference such as this one, or accept [III] as a conclusion of [VII] and [VIII] in this case. In his view, this is a piece of evidence that the human mind does not follow logic and does not work using logical forms. Furthermore, what happens in this example is very easy to explain from the frameworks of MM and UMM.

According to these last approaches, sentences are not linked to logical forms such as [I], but to iconic possibilities standing for circumstances in the world. Reasoning consists of combining such iconic possibilities to come to conclusions. Conditionals often refer to iconic possibilities -not formulae- with structures similar to the disjuncts of [IV]. In those cases, Modus Tollendo Tollens can be applied for reasons akin to the ones indicated.

Let us suppose a conditional such as 'if this is a book, then that is a journal'. Its iconic possibilities have the structure of the disjuncts of [IV] ($p \wedge q$, $\neg p \wedge q$, and $\neg p \wedge \neg q$). However, they are not formulae. The models iconically describe the possible circumstances to which the conditional

refers. They are the possibility in which 'this is a book and that is a journal', the possibility in which 'this is not a book and that is a journal', and the possibility in which 'this is not a book and that is not a journal'. Because the second premise in a hypothetical inference of Modus Tollendo Tollens with this conditional would be 'that is not a journal', the first and the second possibilities have to be rejected: 'that is a journal' in them. The only possibility is hence the third one. But, in that one, 'this is not a book'. Accordingly, this last information is the only one that can be inferred and can be the conclusion of the inference.

However, following Johnson-Laird (2010), that is not the case of the inference taken as an example above. In this last inference, the first premise does not refer to the three possible situations indicated, but only to these two possibilities:

[IX] (She played a musical instrument) & (She did not play a flute)

[X] (She did not play a musical instrument) & (She did not play a flute)

As said, [IX] and [X] are not formulae but iconic models representing reality. Therefore, they are built by virtue of the semantic content of the sentence and the possible pragmatic factors that can have an influence on it. Thus, it is not possible that she did not play a musical instrument and she played a flute at the same time: a flute is a musical instrument. So, the third possibility that would habitually exist for a conditional, the possibility in which both the antecedent and the consequent are false, is removed here. This in turn explains why people do not derive the conclusion in cases such as this one. There is no scenario in which what is indicated by the second premise is true, that is, in which the consequent of the conditional is false (both in [IX] and in [X] is true). Hence, that situation is an impossible situation in the context described; it can happen in no way.

There are many differences between MM and UMM. Nonetheless, what has been explained is common to both of them. As far as the goal of this paper is concerned, the main difference refers to the way the models or possibilities are linked in each case. In MM, the relation is disjunctive (see, e.g., Khemlani et al., 2018). Accordingly, models such as [IX] and [X] can be linked in this way: [IX] or [X]. On the other hand, in UMM, the link is the one of a 'conjunction of possibilities' (see, e.g., Khemlani et al., 2018). Hence, those very models can be related as follows: Possible [IX] and Possible [X].

The theory has received several criticisms. For the particular case of UMM, there are different recent papers challenging it. For example, it

has been said that its conjunctions of possibilities reflect cases in truth tables (e.g., Baratgin, Douven, Evans, Oaksford, Over, Politzer, & Thompson, 2015), that the proponents of the theory tend to speak about logic as if logic were just one system, and there were not diverse kinds of logics (e.g., Bringsjord & Govindarajulu, 2020), or that the assumption of certain theses of UMM can lead to absurd conclusions form the logical point of view (e.g., Oaksford, Over, & Cruz, 2019). However, regarding this, two points can be made. First, the mental model theorists have tried to respond to most of these criticisms (e.g, Johnson-Laird, Khemlani, & Goodwin, 2015b; Johnson-Laird & Ragni, 2019; López-Astorga, Ragni, & Johnson-Laird, 2021). Second, in many cases, as shown below, those criticisms do not have an influence on the arguments of the present paper.

Thereby, what is important now is that, according to both MM and UMM, the inferential process described above is an example of the real manner people reason. The theory often claims that manner has no relation to logical forms. Nevertheless, following López-Astorga, some links are possible.

López-Astorga's proposal

The proposal given by López-Astorga is not only described in the chapter indicated. There are also other works in which it is explained (some of them are cited in that very chapter). Therefore, I will base on the general lines of all of those works in this section.

A very important point in his proposal is that models such as [IX] and [X] can be deemed as conjunctions. Actually, this is an idea that is also to be found in texts written by proponents of MM (e.g., Quelhas et al., 2017, p. 1004). However, what is truly interesting now is that this allows transforming [IX] and [X] into:

[XI] $p \wedge \neg q$

[XII] $\neg p \wedge \neg q$

Obviously, [XI] corresponds to [IX] and [XII] to [X]. Nevertheless, López-Astorga takes another step forward and links formulae such as [XI] and [XII] by means of disjunctions. And this is the point in which the most important difference between UMM and López-Astorga's proposal appears to be. MM also relates models by means of disjunction. However, UMM considers a set of models such as [IX] and [X] to be a conjunction of possibilities

(see also, e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). But precisely because they are possibilities, López-Astorga continues to link formulae such as [XI] and [XII] by means of disjunctions. This enables to come to other formulae whose truth tables can reveal the real logical form of sentences. For example, it is evident that the habitual logical form of conditional sentences such as 'if this is a book, then that is a journal' is [I]. This is because, as it can be deduced from what has been said, their iconic possibilities can be transformed into the disjuncts of [IV], and linked by means of disjunctions to obtain [IV]. Thus, as shown in [V], the truth-values of [I] match those of [IV]. But the case of expressions such as that of the example offered by Johnson-Laird (2010) mentioned above is different. The first premise includes the words 'if' and 'then', that is, the words usually assigned to conditional sentences. However, its actual possibilities are [XI] and [XII]. Therefore, the formula that can be built from its models is:

[XIII] $(p \wedge \neg q) \vee (\neg p \wedge \neg q)$

Which does not allow thinking about logical forms such as [I], but such as, for example,

[XIV] $(p \vee \neg p) \wedge \neg q$

Or just [II].

And the reason for this is obvious too, since, as it is well known,

[XV] $v(\text{XIII}) = v(\text{XIV}) = v(\text{II})$

[XV] reveals that the truth-values of [XIII], [XIV], and [II] are the same. This in turn shows that the true logical form of the first conditional sentence in the example taken from Johnson-Laird (2010, p. 201) has to be akin to formulae such as [XIV] or [II]. Nevertheless, maybe the most relevant point here is that, if this is that way, then the premises of that example are not actually [VII] and [VIII], but [XIV] and [VIII], or [II] and [VIII]. There is no doubt that to infer [III], both from [XIV] and [VIII] and from [II] and [VIII], continues to be possible. In logic, Ex *Contradictione Quodlibet Sequitur* principle allows deriving any conclusion from a contradiction, and a contradiction exists in the two cases. [VIII] is incompatible with the second conjunct of [XIV]. Likewise, [VIII] is the opposite of [II]. However, for the same reasons, in both cases it is also possible to deduce the negation of [III], that is,

[XVI] p

Given [XIV] and [VIII], because [XIV] includes [II], and [II] is the negation of [VIII], we can infer [III]. But that allows us to infer [XVI] too. In the same way, given [II] and [VIII], because they are contrary formulae for each other, we can infer [III]. But that allows us to infer [XVI] as well. So, the final result is that, in any case, both [III] and [XVI] can be inferred, and this can easily explain why individuals reject inferences such as that of Johnson-Laird (2010, p. 201). If both a formula and its negation ([III] and [XVI]) can be derived from the premises (whether they are [XIV] and [VIII], or [II] and [VIII]), it is obvious why people tend not to admit inferences of that kind (for similar arguments on contradictions in other theoretical contexts, see, e.g., López-Astorga, 2017b).

In this way, it can be said that this is a syntactic or formal explanation of the same phenomenon described by Johnson-Laird. Therefore, it can also be stated that the links between semantics and syntax, between iconic semantic models and syntactic logical forms, are not impossible in the theory of mental models. The previous account assumes an important part of the core theses of the theory and, despite that, it can relate the semantic explanation of the theory of mental models to formal structures, rules, and requirements of classical logic.

Of course, this is only an instance. According to works such as those of López-Astorga quoted above, a similar relation between the semantic models of the theory of mental models and the logical forms of propositional calculus can be provided in all of the cases in which individuals come to conclusions apparently contrary to this last calculus. If this last hypothesis is correct, it can be claimed, in the same manner, that there are parallel syntactic explanations based on logical forms for all of the semantic explanations of the theory. Nevertheless, another important idea in works such as those of López-Astorga (2017a) is that it is true that people do not tend to start reasoning doing what logic indicates or enables. As accounted for by the proponents of both MM and UMM, individuals often begin to reason following the information contained in semantic iconic models, which, as mentioned, can also be modified by pragmatics. However, the final conclusions obtained by individuals can be consistent with logic. If the apparent logical forms of sentences are ignored and the real ones are recovered in the way described above, it is possible to note, in all the cases, that their inferential activity can be coherent with calculi similar to the propositional one.

Nonetheless, from the point of view some adherents of the theory of mental models hold, explanations such as the one of López-Astorga cannot be accepted. The reasons are several, one of them being that the possibilities in the conjunctions of possibilities of UMM cannot be related to truth tables

(e.g., Johnson-Laird & Ragni, 2019). Actually, this idea can be correct, since the possibilities are conjuncts, and the conjuncts have to be all true in a conjunction. This is not what happens in a truth table, where the different cases can be true at the same time in no way (e.g., Johnson-Laird & Ragni, 2019). Nevertheless, this objection is easy to overcome. López-Astoga's proposal is not that the iconic models on the human mind are rows in truth tables. His idea is that, from those models or possibilities, truth tables can be built. Thus, the conjunctions of possibilities are not truth tables. The truth tables are constructed a posteriori to provide links between logic and the theory of mental models. However, the most difficult arguments to be addressed from López-Astorga' approach may be already in papers such as that of Johnson-Laird (2010). The next section analyzes those arguments and tries to respond to them under López-Astorga's framework.

The arguments provided in Johnson-Laird (2010)

Johnson-Laird (2010) acknowledges that a syntactic account can be given. It can be argued that people know the definitions of both 'musical instrument' and 'flute'. Hence, they can note that the first sentence of the inference cited can be related only to two possible scenarios:

> Either she played a musical instrument and didn't play a flute or she didn't play a musical instrument and didn't play a flute (Johnson-Laird, 2010, p. 203).

Thus, following him, this logical form could be identified:

> A or else not-A, and not-B (Johnson-Laird, 2010, p. 204).

However, according to him, to assume this idea has three problems. First, to speak about possibilities is, at least in a sense, to previously accept basic theses of the theory of mental models. Second, if we identify possibilities and can already reason from them, logical forms are not necessary. Third, given the logical form 'A or else not-A, and not-B' and the formula 'B', it is possible to derive, in classical logic, 'not-A' too. This last point means something indicated in the previous section as well: given, on the one hand, the formulae [XIV] or [II] and, on the other hand, the formula [VIII], it would continue to be possible to infer [III].

Although I think that the arguments above already respond these three objections in a more or less explicit way, I will address them below again. However, before that, I will make some clarifications, which refer

to the fact that Johnson-Laird criticism does not seem to have an influence on López-Astorga's account. What Johnson-Laird appears to mean is that, from a formal perspective, it could be held that people transform the conditional of the inference into an expression such as 'Either she played a musical instrument and didn't play a flute or she didn't play a musical instrument and didn't play a flute'. From that expression and by means of a process that is a mystery, they deduce a logical form such as 'A or else not-A, and not-B'. Nevertheless, this is not what is proposed by López-Astorga. What this last author claims is that the account of the theory of mental models is, in general, right. However, the iconic models can be transformed into well-formed formulae of propositional calculus, and the truth tables of those well-formed formulae can lead to the real logical forms of sentences. So, individuals do not necessarily make that transformation and become aware of the actual logical forms. López-Astorga's idea is only that it is possible, even if that is made a posteriori and by a person with logical training, to identify an underlying syntax in the framework of the theory of mental models. Therefore, as pointed out, it is not that people make inferences using rules or based on requirements of classical logic, but only that their final conclusions are consistent with such rules and requirements. It can be said that human reasoning is logical just in this way.

Nonetheless, beyond these commentaries, the three problems Johnson-Laird (2010) raises are not hard to answer from López-Astorga's perspective. As far as the first one is concerned, in several works, and not only in those cited here, López-Astorga explicitly acknowledges that his proposal assumes the basic theses of the theory of mental models. In fact, the real idea is that reasoning is as explained by theory. What is added is only that the account of the theory of mental models is not absolutely incompatible with logical forms; it is always possible to explain in a syntactic or formal way what MM and UMM often explain resorting only to semantics (and pragmatics). Thus, it is not a difficulty for López-Astorga's approach that it includes essential ideas of the theory of mental models. He considers most of the theses of this last theory to be right and assumes that people reason as its proponents indicate.

Thus, the second one is not a real problem either. The key is not whether logical forms are necessary or not. Probably, they are not. As shown by the adherents of the theory of mental models, the human mental activity can be explained in an easy way without them. The point is that the fact that it is possible to recover deep logical forms based on the iconic possibilities of theory reveals that reasoning is not contrary to logic. Logical forms may

not be necessary, but it does not mean that there are not any relations to classical propositional calculus. In fact, it can be assumed that, because these last relations are possible, human thought is consistent. Accordingly, the idea is that, while semantics and pragmatics play a very important role in human reasoning, a role the theory of mental models clearly describes, that role does not prevent an underlying syntax in the mental processes from being identified.

Finally, the third objection has been clearly answered in the previous section. In classical logic, 'not-A' can be inferred from 'A or else not-A, and not-B' and 'B'. By means of Ex Contradictione Quodlibet Sequitur principle, that can be made. Nonetheless, exactly in the same way, 'A' can be derived too. So, if it is possible to deduce both 'not-A' and 'A', it is not clear, given the premises 'A or else not-A, and not-B' and 'B', which of those two formulae is the suitable conclusion, since 'A' is the opposite of 'not-A' (it is its negation). Furthermore, this is a problem that has several solutions. For example, there is a theory rival to the one of mental models: the mental logic theory. It claims that people directly detect logical forms in a natural way and apply formal schemata to them. To solve difficulties of this kind, the mental logic theory argues that the actual mental logic is not as standard logic, and that contradictions do not work in the same way in those two systems. In the former, Ex Contradictione Quodlibet Sequitur principle does not exist and contradictions do not allow inferring any formula. They only show that some of the data assumed are false (e.g., Braine & O'Brien, 1998).

Conclusions

My conclusions here match those indicated by López-Astorga in works such as those quoted in this paper. It is true that, if we accept the theses of the theory of mental models, we do not need logical forms to explain the way human beings reason. However, if what we wish is to understand human cognition in its full complexity, it may not be superfluous to try to demonstrate, as López-Astorga does, that the inferential processes that theory describes can be linked to syntactic derivations valid in classical logic.

Perhaps our intellectual activity is fundamentally semantic. Nevertheless, that does not imply that it is in conflict with any type of syntax, and that it has no links to formal structures. It is clear that the system presented by Gentzen (1934, 1935) does not lead our reasoning. Maybe the greatest mistake made in the experimental cognitive studies for some time was to admit the opposite assumption. Nevertheless, it can be another mis-

take to suppose that the human mind has no connection to that system or one like it.

In this way, López-Astorga's proposal seems to suggest that, although individuals do not make everything a system of that kind allows, their inferential conclusions appear not to be incoherent with its requirements. This is a line that can continue to be explored, since it can reveal which the real relations between semantics (and pragmatics) and syntax are. It seems no mere coincidence that the models of MM and UMM can be related to logical forms. It is also interesting that the conclusions that can be obtained with those logical forms are very akin to those to which the models lead. In addition, in ideal circumstances, that is, when semantics or pragmatics do not modify the initial models, those models coincide with the cases in which the connective is true in its logical truth table (for a more detailed discussion in this regard, see, e.g., López-Astorga, 2017a). So, one might think that this is a line of research that is not depleted.

Accordingly, as it can be inferred from all that has been said in this paper, the basic idea would be essentially the same as that of López-Astorga. There is no doubt that the theory of mental models rightly accounts for, in general, the way the human mind works. Nonetheless, the acceptance of this fact and hence of the theory as the best theory explaining reasoning does not imply a rejection of logical forms. That only requires noting that such forms are not the main basis of the mental activity. However, we can continue to deem those forms as one of the several aspects of that activity. Although it is not the most relevant, it can deserve to be analyzed and studied in detail as well.

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