A BILATERAL REDUCTION SENTENCE FOR MODULATION*

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Abstract:
The theory of mental models considers sentential connectives to refer to semantic possibilities. The theory also proposes that the possibilities can be amended by virtue of modulation processes, that is, processes in which semantics or pragmatics can have an influence. The definition of modulation the theory of mental models gives has been deemed as a scientific definition within Carnap's framework. This is because it is easy to find reduction sentences corresponding to it. What seems to be harder is to think about bilateral reduction sentences for that definition. This paper addresses this issue. It shows that at least a bilateral reduction sentence is possible for modulation.

Key words: bilateral reduction sentence; modulation; possibility; reduction sentence; theory of mental models.

Introduction

A very important concept in cognitive science in general and in the theory of mental models in particular is that of modulation (e.g., Quelhas & Johnson-Laird, 2017; Quelhas, Johnson-Laird, & Juhos, 2010). According to that theory, the interpretation of language consists of the identification of semantic possibilities or models associated with each sentence (see also, e.g., Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999). But semantics and pragmatics can alter the initial possibilities of a sentence. This can happen because of modulation. Modulation can adjust and even remove possibilities (see also, e.g., Byrne & Johnson-Laird, 2020).

The general definition of modulation the theory of mental models offers is as follows:

"The process in the construction of models in which content, context, or knowledge can prevent the construction of a model and can add information to a model" (Johnson-Laird, Khemlani, & Goodwin, 2015, p. 202).

Checking whether or not (1) is a scientific definition has been attempted. To do that, Carnap's (1936, 1937) general framework has been assumed. The result has been positive: reduction sentences (in particular, three reduction sentences) can be built for (1) (López-Astorga, 2022).

The issue that the present paper addresses is the fact that Carnap (1936) not only speaks about reduction sentences, but also deals with reduction pairs and bilateral reduction sentences. The three reduction sentences proposed in the literature for modulation are not problematic. But it is important

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to analyze whether or not the essential concepts involved in (1), that is, content, context, and knowledge, allow constructing reduction pairs and bilateral reduction sentences too. The aim here is to show that this is possible. Therefore, the paper tries to argue that the definition of modulation in the theory of mental models is scientific in accordance with Carnap's (1936, 1937) different criteria.

To achieve that goal, this paper will have several sections. One of them will describe the way Carnap (1936) considers scientific definitions. This will be explained by means of the concepts of reduction sentence, reduction pair, and bilateral reduction sentence. Then, how the theory of mental models understands modulation will be accounted for. The next section will show the reduction sentences that have been proposed for modulation. One more section will point out the objections that can be offered for those sentences based on the manner in which Carnap (1936) presents reduction pairs and bilateral reduction sentences. Finally, how the objections can be overcome will be indicated. This will allow for the claim that the definition of modulation in the theory of mental models is scientific and progressively confirmable in an even stronger sense than that sustained in the literature.

Rudolf Carnap and scientific definitions

Carnap (1936) seems to agree with Popper (2002) about the impossibility of absolutely verifying sentences. For that reason, Carnap (1936) presents reduction sentences, reduction pairs, and bilateral reduction sentences in order to gradually confirm whether or not particular properties can be associated with particular definitions. Thus, to know the properties that can correspond to a predicate R, a sentence such as (2) can be built.

$$\forall x\ [Px \rightarrow (Qx \rightarrow Rx)],$$

where '∀' is the universal quantifier, P and Q are predicates, and '→' represents a conditional relation.

According to Carnap (1936), (2) is 'a reduction sentence for R', since it enables one to check to what extent predicates such as P and Q can be linked to R. But this can be only if requirement (3) is fulfilled.

$$\neg\forall x\ \neg(Px \land Qx),$$

where '¬' is negation and '∧' stands for conjunction.

Requirement (3) provides that cases of P and Q at the same time have to exist.

Reduction sentence (2) is not the only kind of sentence that can be constructed to know what the properties that can be attributed to R are. Following Carnap (1936), it is also possible to build a reduction pair such as that consisting of (2) and (4).

$$\forall x\ [Sx \rightarrow (Tx \rightarrow \neg Rx)],$$

where S and T are predicates.

Sentences (2) and (4) are 'a reduction pair for R'. This reduction pair not only allows one to review whether or not P and Q can be related to R. It also enables one to analyze whether or not S and T are properties incompatible with R as well. Carnap (1936) provides a requirement for reduction pairs too:

$$\neg\forall x\ \neg[(Px \land Qx) \lor (Sx \land Tx)],$$

where '∨' indicates disjunction.

Requirement (5) claims that either at least a case of P and Q at once or a case of S and T at once should exist.

Lastly, a bilateral reduction sentence such as (6) can also be built.

$$\forall x\ [Px \rightarrow (Qx \leftrightarrow Rx)],$$

where '↔' expresses a biconditional relation.

Sentence (6) is 'a bilateral reduction sentence for R'. What (6) tries to confirm is whether or not the link between Q and R is much stronger than in (2). Carnap (1936) gives a requirement here as well:

$$\neg\forall x\ \neg Px.$$

Requirement (7) points out that at a minimum a case of P is necessary.

Reduction sentences have been offered for definition (1). But it appears that the construction of reduction pairs or bilateral reduction sentences from those reduction sentences can be controversial. This issue will be developed below. An important point in the argumentation will be related to the fact that, as Carnap (1936) acknowledges, (6) gets derived from (2) and (4) if (8) is the case.

$$\forall x\ [(Px \leftrightarrow Sx) \land (\neg Qx \leftrightarrow Tx)].$$

Sentence (8) serves to transform (4) into (9).

$$\forall x\ [Px \rightarrow (\neg Qx \rightarrow \neg Rx)]$$

and (6) can be inferred from (2) and (9).

This point will be essential for the objection that can be proposed regarding the three reduction sentences that have been offered for (1). Before showing this, the next section explains how modulation works in the theory of mental models.

Modulation in the theory of mental models

According to the theory of mental models, modulation can have an influence on any sentential connective (see also, e.g., Johnson-Laird, 2012). But given that the example used in the literature to present the reduction sentences for (1) is disjunction (López-Astorga, 2022), the present paper will only consider, as an instance, this last connective. However, it is important not to forget that the account would be similar for any other sentential connective.

In principle, a disjunction such as (10) would have three possibilities or models (see also, e.g., Quelhas, Rasga, & Johnson-Laird, 2019).

Either they eat salad or they eat fries

The possibilities or models would represent situations in which (10) would be true (see also, e.g., Johnson-Laird & Ragni, 2019). Those possibilities or models would be (11), (12), and (13).

It is possible that they eat salad and they eat fries

It is possible that they eat salad and they do not eat fries

It is possible that they do not eat salad and they eat fries

Although cases (11), (12), and (13) seem to reveal that the theory of mental models is not different from logic, it is. Many features distinguish the theory of mental models from logic (see also, e.g., Espino, Byrne, & Johnson-Laird, 2020; Johnson-Laird, 2010; Khemlani, Hinterecker, & Johnson-Laird, 2017; López-Astorga, Ragni, & Johnson-Laird, 2022). To explain all of them would move this paper away from its aim. Only one characteristic differentiating the theory from logic is important here: modulation.

Modulation in the theory of mental models is defined as (1). Hence, it can happen by virtue of content, context, and knowledge. In other words, content, context, and knowledge can modify models or possibilities. (The rest of this section is based on the account, the examples, and the models or possibilities assigned to those examples in López-Astorga, 2022.)

The case of content can be the following:

"Either you eat rice or you eat rice and chicken" (López-Astorga, 2022, p. 112).

While it is a disjunction, sentence (14) only enables two possibilities: (15) and (16).

It is possible that you eat rice and you do not eat chicken

It is possible that you eat rice and you eat chicken

If (14) is true, there is no other possibility. This fact has to do with content. In any context in which sentence (14) is said, people can note that (15) and (16) are the only possibilities, even if they do not know what rice and chicken are. The word 'rice' appears in the two disjuncts. That implies that to know its meaning is not necessary in order to be aware that the situation in which you eat rice is always the case. So, neither context nor knowledge plays a role here.

As far as context is concerned, an example can be that of the following circumstance (taken from López-Astorga, 2022): two people read a menu indicating that they can choose an apple or an orange for dessert. If after eating an apple, they ask whether they can also take an orange, what (17) means is that they cannot.

"Either you eat an apple or you eat an orange" (López-Astorga, 2022, p. 114).

In the context described, given that they have already eaten an apple, the only model or possibility that can be attributed to (17) is (18).

It is possible that you eat an apple and you do not eat an orange

This context restricts the models or possibilities. The content would enable more models or possibilities without it. Regarding knowledge, its action is not present in this case either. What (17) implies as an answer in the context is not directly related to the meaning of the words 'apple' and 'orange'. If the two alternatives for dessert were different, the situation would not be distinct.

Finally, the example for knowledge can be (19).

"Either Pat visited Milan or she visited Italy" (Johnson-Laird et al., 2015, p. 204; see also López-Astorga, 2022, p. 108).

Now, the models or possibilities are (20) and (21).

It is possible that Pat visited Milan and she visited Italy

It is possible that Pat did not visit Milan and she visited Italy

Pat cannot be in Milan without being in Italy.

So, there are no more possibilities. And this is only because of knowledge. The possibilities for (19) are (20) and (21) in any context (19) is said. On the other hand, content does not have an influence in the sense it has in (14).

From these examples, reduction sentences can be offered. The next section will show that. But it is important to remember that only content causes modulation in (14) (neither context nor knowledge), only context causes modulation in (17) (neither content nor knowledge), and only knowledge causes modulation in (19) (neither content nor context).

Modulation and its reduction sentences

Given (1) and the fact that content, context, and knowledge can, separately, modulate sentences, to construct reduction sentences corresponding to the definition of modulation is easy. (The rest of this section will also follow López-Astorga's, 2022, account and its reduction sentences.)

It is enough to assume these equivalences:

P: To be a sentence

R: To be modulated

U: To have content modifying models or possibilities

V: To be said in a context modifying models or possibilities

W: To be related to knowledge modifying models or possibilities

In this way, these three reduction sentences can be built:

$$\forall x\ [Px \rightarrow (Ux \rightarrow Rx)]$$

$$\forall x\ [Px \rightarrow (Vx \rightarrow Rx)]$$

$$\forall x\ [Px \rightarrow (Wx \rightarrow Rx)]$$

Formulae (22), (23), and (24) capture the three cases in which modulation can happen. For this reason, it can be said that they are reduction sentences for R. Even (25) could be claimed.

$$\forall x\ [Rx \leftrightarrow (Ux \vee Vx \vee Wx)]$$

This is essentially the account that can be found in the literature (López-Astorga, 2022). It seems to reveal that the definition of modulation is coherent with the requirements Carnap (1936, 1937) provides to construct scientific definitions that can be gradually confirmed. But, if reduction pairs and bilateral reduction sentences are considered, all of this appears to be more complex. The reason will be shown below.

Are the reduction sentences for modulation incompatible?

The main difficulty (22), (23), and (24) present is that they can include mutually exclusive predicates. (22) is the reduction sentence corresponding to (14). (14) is a sentence in which the action of modulation is just by virtue of content (with no role for context or knowledge). That implies (26).

$$\exists x\ (Px \wedge Ux \wedge \neg Vx \wedge \neg Wx),$$

where '∃' is the existential quantifier.

Formulae (22) and (26) suffice to lead to Rx. But this does not allow one to construct a bilateral reduction sentence. (22) and (27) cannot be a reduction pair for R enabling the derivation of (28).

$$\forall x\ [Px \rightarrow (\neg Ux \rightarrow \neg Rx)]$$

$$\forall x\ [Px \rightarrow (Ux \leftrightarrow Rx)]$$

Sentences (23) and (24) show the falsity of (27) and, accordingly, (28).

Something similar occurs with (23). It is the reduction sentence corresponding to (17). In (17), context is the only component acting for modulation. Hence, given (17), (29) can be accepted.

$$\exists x\ (Px \wedge Vx \wedge \neg Ux \wedge \neg Wx)$$

Again, (23) and (29) are enough to conclude Rx. But a bilateral reduction sentence is hard here too. (23) and (30) are not a reduction pair for R.

$$\forall x\ [Px \rightarrow (\neg Vx \rightarrow \neg Rx)]$$

As in the previous case, (22) and (24) prevent (30) and (31) from being true.

$$\forall x\ [Px \rightarrow (Vx \leftrightarrow Rx)]$$

And the same happens with (24). This last sentence can be linked to (19). Knowledge is what modulates in (19), without influence from content or context. Therefore, (32) should be assumed.

$$\exists x\ (Px \wedge Wx \wedge \neg Ux \wedge \neg Vx)$$

Although (24) and (32) allow one to deduce Rx, once again, a bilateral reduction sentence is difficult. (24) and (33) cannot be accepted as a reduction pair for R.

$$\forall x\ [Px \rightarrow (\neg Wx \rightarrow \neg Rx)]$$

This is because (22) and (23) show that (33) and (34) are false.

$$\forall x\ [Px \rightarrow (Wx \leftrightarrow Rx)]$$

So, it can be stated that, given the definition of modulation in (1), it is possible to provide reduction sentences such as (22), (23), and (24). Nevertheless, this seems to be different in the case of bilateral reduction sentences. It appears that bilateral reduction sentences linked to (1) are hard. If this is the case, the definition of modulation the theory of mental models gives is weak within Carnap's (1936, 1937) framework, at least in a sense: it does not allow for reference to bilateral relations. But this is not correct. The next section tries to show that.

Building a bilateral reduction sentence for modulation

Although it may seem to be the opposite, reduction pairs are possible as well. For example, (4) could be taken along with (22), (23), or (24) if (35) holds.

$$\forall x\ \{(Px \leftrightarrow Sx) \wedge [Tx \leftrightarrow (\neg Ux \wedge \neg Vx \wedge \neg Wx)]\}$$

To consider predicates U, V, and W at once reveals that a bilateral reduction sentence is also possible. (25) gives an important clue too. By virtue of (35), (4) can be transformed into (36).

$$\forall x\ \{Px \rightarrow [(\neg Ux \wedge \neg Vx \wedge \neg Wx) \rightarrow \neg Rx]\}$$

and (36) is equivalent to (37).

$$\forall x\ \{Px \rightarrow [\neg(Ux \vee Vx \vee Wx) \rightarrow \neg Rx]\}$$

Therefore,

$$\forall x\ \{Px \rightarrow [Rx \rightarrow (Ux \vee Vx \vee Wx)]\}$$

On the other hand, if, for example, (22) is true, (39) is too.

$$\forall x\ \{Px \rightarrow [(Ux \vee Vx \vee Wx) \rightarrow Rx]\}$$

In fact, (39) is also true in the cases in which (23) and (24) are, respectively, true. Hence, (38) and (39) lead to bilateral reduction sentence (40).

$$\forall x\ \{Px \rightarrow [(Ux \vee Vx \vee Wx) \leftrightarrow Rx]\}$$

or, if preferred,

$$\forall x\ [Px \rightarrow (\neg Tx \leftrightarrow Rx)]$$

Sentences (40) and (41) can be deemed as bilateral reduction sentences for R, or definition (1).

Conclusions

In the literature, it has already been shown that it is possible to build reduction sentences for the definition of modulation the theory of mental models provides, that is, for (1). In particular, given that the characteristics that can modulate the models or possibilities of a sentence are three (content, context, and knowledge), three reduction sentences can be constructed, each of them capturing one of those features. They are sentences (22), (23), and (24).

The problem is that, as also described in the literature, those reduction sentences can be linked to sentences in natural language having one of the three characteristics and excluding the other two characteristics, that is, to sentences such as (14), (17), and (19). Thus, one can think about sentences modulated by virtue of content, and in which context and knowledge play no role, for example, (14). Likewise, there are sentences modulated by virtue of context, and on which content and knowledge have no influence, for example, (17). In the same way, it is possible to find sentences modulated by virtue of knowledge, and in which content and context do not act, for example, (19).

This is important because it seems to imply that bilateral reduction sentences are not possible for the definition of modulation in the theory of mental models. If each of the features that can cause modulation can exclude the other two features, biconditional relations cannot be established.

Nonetheless, a solution can be presented. If the three characteristics are negated at once and linked by means of conjunctions, as in (35) and (36), it is possible to build reduction pairs. Those reduction pairs can lead to a bilateral reduction sentence: (40), or (41).

It is not possible to construct bilateral reduction sentences for each of the characteristics that can modulate a sentence. This is because those characteristics, in certain cases, can imply that the other two characteristics are not present. But, assuming that the relation between the characteristics is disjunctive, that disjunction can be, as in (37), (38), and (39), taken as an only clause. This allows building a bilateral reduction sentence. Accordingly, the definition of modulation in the theory of mental models fits Carnap's (1936, 1937) framework even better than indicated in the literature.

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