THE RECOVERY OF SYNTACTIC STRUCTURES CANNOT IGNORE TEMPORALITY

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Abstract: *Although its proponents do not support that idea, the theory of mental models has been used to recover deep syntactic structures of sentences. In fact, the theory has been linked to System K and modal logic, and hidden logical forms with modal operators have been proposed. However, this paper analyzes one of the last developments of the theory, the one related to counterfactual disjunctions and conditionals, in order to check whether or not that methodology can also be applied to these two kinds of sentences. The result seems to show a negative conclusion, that System K is not enough in the case of counterfactual sentences, and that, if it is wanted to recover the syntactic structures of these last sentences, it is necessary to resort, at least, to temporal logical elements as well.*
Keywords: *conditional, disjunction, mental model, possibility, syntactic structure.*

Introduction

The theory of mental models (e.g., Khemlani& Johnson-Laird, 2019) is a current cognitive proposal accounting for reasoning and human communication based on mainly semantics and pragmatics, and putting hence syntax in second place (see also, e.g., Quelhas& Johnson-Laird, 2017). However, despite that, the methodology of the theory has been used to find the real underlying syntactic structures corresponding to linguistic expressions too (e.g., López-Astorga, 2018). So, it can be said that, beyond its proponents' goals, the theory of mental models can have important consequences both in logic and in linguistics.

In particular, something made in this way is to link the possibilities that, following semantic and pragmatic criteria, the theory of mental models assigns to sentences to System K of modal logic (e.g., López-Astorga, 2018). And this paper will continue in this direction. Nevertheless, the potential of

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the theory to describe human mental activity will not be addressed here. Only the resources it can give to identify syntactic forms from the exclusively linguistic perspective will be dealt with below. Thus, with this aim, one of the more recent developments of the theory of mental models will be reviewed. That development is related to counterfactual disjunctions and conditionals and is basically presented by Byrne and Johnson-Laird (2019).

This will allow showing the point of this paper. That point is that, to detect real syntactic structures, it is not enough to link the possibilities of the theory of mental models to System K. In many cases, for example, when the expressions refer to counterfactual disjunctions and conditionals, it is required to resort to additional logical machinery capturing temporal differences. Of course, this idea is not absolutely new, because it has already been raised from other perspectives (see, e.g., López-Astorga, 2019). However, what does seem to be new is the study of the role that certain counterfactual sentences can play in this regard, and, as said, that is the main issue to consider here.

To achieve such goals, first, a brief description of the recent developments of the theory of mental models about counterfactual disjunctions and conditionals will be offered. Then, the general lines of frameworks relating the aforementioned theory to System K will be described. Lastly, it will be argued why the way that very theory understands counterfactual sentences reveals that System K is insufficient to detect all of the possible formal structures of expressions, and that, in several situations, at a minimum, elements of a system of temporal logic are also necessary.

The theory of mental models and counterfactual disjunctions and conditionals

Perhaps one of the most basic ideas of the theory of mental models is that sentences are linked to possibilities that can be derived from them by virtue of basically semantic and pragmatic criteria (see also, e.g., Khemlani, Byrne, & Johnson-Laird, 2018). Nevertheless, the possibilities of, for instance, a simple inclusive disjunction expressed without context are easy to identify (see also, e.g, Johnson-Laird, 2012). Indeed, the possibilities of a disjunction such as this one are clear:

(1) "There is beer, or there is wine, or both" (Byrne & Johnson-Laird, 2019, p. 2).

Following Byrne and Johnson-Laird (2019), who in turn refer to Khemlani et al. (2018), the possibilities corresponding to (1) are:

(2) Possible (p & ¬q) & Possible (¬p & q) & Possible (p & q)

Where 'p' stands for the first disjunct, 'q' represents the second one, and '¬' indicates negation.

The possibilities in (2), which remind the cases in which an inclusive disjunction is true in a truth table of classical logic, are expressed in a way akin (although not exactly identical) to the one used by Byrne and Johnson-Laird (2019). Nonetheless, maybe two points are those that are important to highlight now. On the one hand, (2), as explicitly pointed out in this last paper, is only incompatible with one hypothetical scenario: when there is neither beer nor wine. On the other hand, what (1) means radically changes when it is expressed in a counterfactual manner:

(3) "There would have been wine, or there would have been beer, or both" (Byrne and Johnson-Laird, 2019, p. 3).

The case of (3) is clearly different, since what it really indicates is, as explained by Byrne and Johnson-Laird (2019), that, currently, there is neither wine nor beer, that, in a past moment in time, there could be wine, that, in a past moment in time, there could be beer, and that, in a past moment in time, there could be both wine and beer. In this way, following also Table 1 in Byrne and Johnson-Laird (2019), it can be claimed that a good manner to express the possibilities of (3) can be (4):

(4) Possible Once (p & ¬q) & Possible Once (¬p & q) & Possible Once (p & q) & Factual (¬p & ¬q)

Something similar happens with the conditional. To draw the possibilities of a simple conditional without contextualizing is very easy, as the ones of (5) are obvious:

(5) "If A happened, then C happened" (Byrne & Johnson-Laird, 2019, p. 3; Table 1).

According to the theory of mental models, the possibilities of a conditional such as (5) are the same as the cases in which a material conditional is true in classical logic, and hence (5) can only be false in one case: when A is true and C is false (see also, e.g., Johnson-Laird, 2012). So, following that point and Table 1 in Byrne and Johnson-Laird (2019), it can be stated that the possibilities corresponding to (5) are:

(6) Possible (A & C) & Possible (¬A & C) & Possible (¬A & ¬C)

However, again, everything changes in the case of counterfactual conditionals such as this one:

(7) "If A had happened, then C would have happened" (Byrne & Johnson-Laird, 2019, p. 3; Table 1).

Now, as also pointed out by Byrne and Johnson-Laird (2019), the message transmitted is more exact. It is said that neither A nor C happened, that, in a past moment in time, both A and C could have happened, that, in a past moment in time, C could have happened without A, and that, in a similar way as (5), the only impossible scenario is that in which A happens and C does not. Therefore, paying attention once again to their Table 1, what (7) actually means can be expressed as follows:

(8) Possible Once (A & C) & Possible Once (¬A & C) & Factual (¬A & ¬C)

But, as said, although that is not its proponents' intention (see, e.g., Johnson-Laird, 2010), there are works linking frameworks such as the one described above to systems such as K (see, e.g., López-Astorga, 2018). The next section shows how this this is done.

The theory of mental models and System K

As it is well known, System K receives its denomination by virtue of Kripke (see, e.g., Kripke, 1963, 1965) and the aspect of it that is specially considered to relate it to the theory of mental models is that, in it, as in all modal logic, the concept of possibility is very relevant as well. Thus, what is done is basically to build possible well-formed formulae from the possibilities identified by using the methodology of the theory of mental models. The proponents of the theory explicitly reject links between it and modal logic too (e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). Nevertheless, in papers such as that of López-Astorga (2018), it is argued that providing relations of that kind are not complex at all.

López-Astorga (2018) uses examples taken from previous works supporting the theory of mental models. Nonetheless, his general idea is easy to understand just paying attention to (1) and (2). He accepts that it is justified to come to (2) from (1). However, that is deemed as just the first step, the second one being to transform (2) into this well-formed formula in System K:

(9) $P(p \& \neg q) \& P(\neg p \& q) \& P(p \& q)$

Where, now, '&' is the logical conjunction, '¬' is the logical negation, and 'P' is the possibility operator in System K, that is, the operator providing that the formula between brackets following it is true in, at a minimum, one possible world (see also, e.g., Fitting & Mendelsohn, 1998).

Of course, this procedure can also be used in the case of (5) and (6). Again, the switch from (5) to (6) can be considered as the first step, the second one being to come to (10).

(10) $P(A \& C) \& P(\neg A \& C) \& P(\neg A \& \neg C)$

In López-Astorga (2018), it is possible to find formulae akin to these ones. In fact, with no exactly the same symbols, (10) is formula [XVI] in López-Astorga (2018). But what appears to be important in this moment is that (10) is also a well-formed formula in System K, and that, as shown, for instance, in López-Astorga (2018), this methodology can be very useful in

the case of much more complicated sentences too. As mentioned, (1) and (5) are simple sentences without context. However, what the theory of mental models really states is that the possibilities are often altered by semantics and pragmatics. For example, the case of this conditional is not that easy:

> (11) "If the workers settle for lower wages then the company may still go bankrupt" (Johnson-Laird & Byrne, 2002, p. 663; Table 4; see also, e.g., López-Astorga, 2018, p. 123).

Although not exactly in the same way and with the same symbols, López-Astorga (2018) points out, following what is argued in Johnson-Laird and Byrne (2002), and, specially, in their Table 4, that the possibilities for (11) are these:

> (12) Possible (p & q) & Possible (p & ¬q) & Possible (¬p & q)

Where, now, 'p' is the antecedent of (11) and 'q' stands for its consequent.

Clearly, (12) makes it evident that a sentence such as (11) is complex. It is a conditional but it admits as a possibility that its antecedent is true and its consequent is false. That is at least what the second possibility in (12) provides. Moreover, it does not enable the possibility of the two clauses being false, that is, what indicates the last possibility in (10).

However, as stated, that is not an obstacle for the use of the methodology presented in papers such as the one of López-Astorga (2018). Undoubtedly, in accordance with that methodology, (12) can be transformed into a well-formed formula in System K such as (13).

> (13) P(p & q) & P(p & ¬q) & P(¬p & q)

With not exactly the same symbols, (13) is formula [XXIII] in López-Astorga (2018). In addition, it is equivalent to (9). Nevertheless, perhaps the most important point about this can be that cases such as (11) are the bases on which the theory of mental models argues that human language and reasoning have nothing to do with classical logic. So, the account by López-Astorga (2018) described in this section can be relevant because it seems to link the essential theses of the theory of mental models, if not to classical

logic, at least, to modal logic. Nonetheless, it also appears that the study by Byrne and Johnson-Laird (2019) raises a new challenge to that account, since, just the machinery provided by System K does not appear to suffice to address counterfactual disjunctions and conditionals such as (3) and (7). And, given that situation, apparently only one solution is suitable: to complement System K by elements capturing the temporality that is present in sets of possibilities such as (4) and (8). This is done in the next section.

Counterfactual disjunctions and conditionals and the logical forms related to them

Indeed, it seems to be impossible to represent by means of only well-formed formulae in System K expressions such as 'Possible Once' in sets such as (4) and (8). As indicated, a temporal dimension can be hardly ignored.

Nevertheless, there are several manners to resort to logical elements expressing temporal difference (see, e.g., López-Astorga, 2019). But, for the particular case being dealt with here, maybe a very simple semantics of moments in time could be enough (for possible characteristics for a semantics of that type and other papers about such characteristics, see also, e.g., López-Astorga, 2019). Such instants, in a similar way as that is done in the literature, could be expressed as follows:

(14) {T1, T2,..., Tn} (see, e.g., again, López-Astorga, 2019, p. 13).

Thus, taking (14) into account, what is indicated by, for example, 'Possible Once (X)', that is, that there was a past moment in which X could happen, could be represented in this manner:

(15) $P(X)_{(Ti)}$

Where Ti< Tc, and 'Tc' is the current moment.

Likewise, an expression such as 'Factual (X)' could be stood for by a formula such as the following:

(16) $X_{(Tc)}$

So, with this new machinery, it would be possible to assign a logical form to (4), which would be:

(17) $P(p \& \neg q)_{(Ti)} \& P(\neg p \& q)_{(Tj)} \& P(p \& q)_{(Tk)} \& (\neg p \& \neg q)_{(Tc)}$

Where, in the same way, Tj< Tc, and Tk < Tc.

And something similar can be claimed for (7) and (8). (8) could be transformed into this well-formed formula:

(18) $P(A \& C)_{(Ti)} \& P(\neg A \& C)_{(Tj)} \& (\neg A \& \neg C)_{(Tc)}$

Which would mean that (18) could be the syntactic structure corresponding to (7), in the same way as (17) could be the one for (3).

As mentioned, maybe there are other ways to denote temporality. However, if something seems to be absolutely clear, that is that sentences such as (3) and (7) cannot have formal structures without references to temporal distances. Evidently, this has a clear consequence: the logical forms or syntactic structures corresponding to counterfactual disjunctions and conditionals cannot be represented just with the formal resources of System K. More elements are necessary, which means that the last developments of the theory of mental models seem to prove that such resources are insufficient to explain human language.

Conclusions

As stated, the theory of mental models tends to dispense with syntax. But, in this paper, without addressing the discussion about whether or not syntax is a secondary element in reasoning (as proposed by that theory), the aim has been just to find the syntactic structures that could be compatible with the last studies carried out under the approach of that very framework. The intention was only to use the resources of the theory of mental models in a sense different from that of its proponents: not to

explain cognition, but to identify formal structures that, from the linguistic point of view, can be attributed to sentences.

From this perspective, the paper followed lines of work such as the one of López-Astorga (20018), which try to relate logical forms of modal System K to the possibilities of the theory of mental models. Nevertheless, that activity showed that just System K is not enough to recover the logical forms of sentences. There are cases in which a temporal dimension is necessary.

Actually, this is not absolutely new. The general idea is already somehow in other papers (see, e.g., López-Astorga, 2019). Nonetheless, what does be new, and that is the contribution of this paper, is that one of the cases in which temporality is clearly needed is the one of counterfactual disjunctions and conditionals.

Therefore, if the theory of mental models is assumed as a correct approach (which is something that, following the specialized literature on that theory and works such as the one of López-Astorga, 2018, seems reasonable to do), it appears to be required to also accept that the logical forms that can correspond to sentences are not only those allowed by a system such as System K. As said, at a minimum, elements indicating temporality are necessary as well. Of course, as also pointed out, to assume the theory of mental models implies to deem syntax as an aspect that is not primary in human cognition. For this reason, it is important to remind that, when the expression 'formal structures' has been used above, the arguments have only referred to logical forms that linguistically can be linked to sentences, whether or not people consider them when making inferences.

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